<< Previous Chapter

Chapter 2

Photometry

In general we must use the well defined and objectively measured quantities of geometrical radiometry in quantitative studies of radiative transfer. However, there are times when, either unavoidably or by choice, the human eye becomes one of our instruments. Such is the case when we wish simply to observe for pleasure the beautiful colors of nature or when, even today, someone uses Secchi disk observations as a semi-quantitative measure of the clarity of a natural water body (Preisendorfer, 1986). In other instances, the eye-brain system may be the preferred instrument, as in visual searches for underwater objects. We therefore require some knowledge of how the human visual system responds to radiant energy. This takes us into the domain of photometry and, more generally, colorimetry. For our present purpose, *geometrical photometry* is defined as the study of the human visual response to the quantities of geometrical radiometry.

2.1 The Photopic Luminosity Function

Not all wavelengths of light evoke the same sensation of brightness in the human eye-brain system. For example, suppose a person with "normal" eyesight is exposed to monochromatic radiance of wavelength 550 nm and magnitude 10^4 W m^{-2} sr^{-1} nm^{-1} (recall from Chapter 1 that this is comparable in magnitude to the sun's spectral radiance at this wavelength). The person will "see" a bright yellowish-green light. If the person is exposed to light of the same radiance, 10^4 W m^{-2} sr^{-1} nm^{-1}, but of wavelength 300 nm, the person will not "see" anything, since the eye is not sensitive to this wavelength in the ultraviolet. However, if the exposure lasts long enough, permanent and severe damage will be done to the eye by the ultraviolet radiant energy.

This *relative ability* of radiant energy to evoke differing sensations of brightness in the human observer is described by the *photopic luminosity function* $\bar{y}(\lambda)$, which is plotted in Fig. 2.1 and tabulated in Table 2.1. This function is an empirically derived composite based on visual response studies of numerous humans. It therefore has the same statistical validity

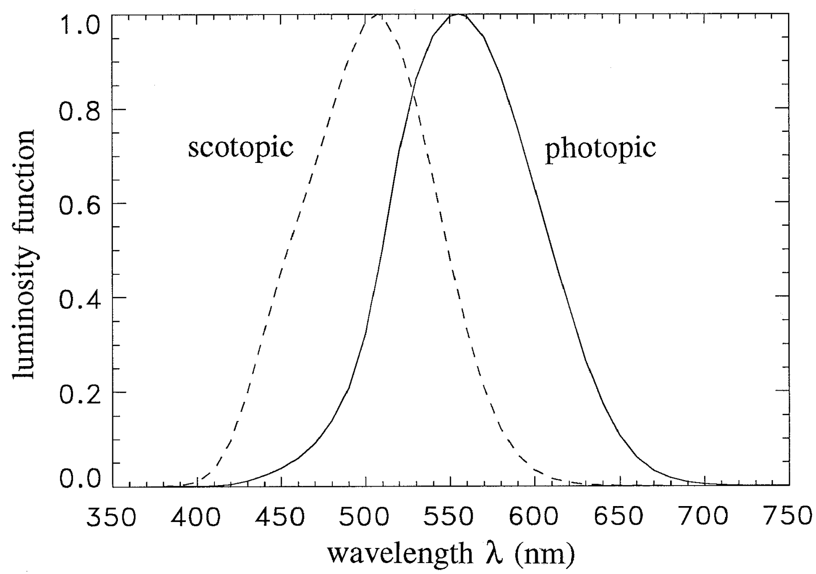

Fig. 2.1. Photopic (bright light) and scotopic (dim light) luminosity functions.

as the "average American male, age 30." Particular individuals can have responses that vary considerably from the mean response. Nevertheless, the function serves as a reasonable *definition* of the eye response of a *standard photometric observer.* Suppose, for example, that monochromatic radiance $L(\lambda{=}500\text{ nm})$ (blue-green light) of some given magnitude (in $\text{W m}^{-2}\text{ sr}^{-1}\text{ nm}^{-1}$) evokes a certain qualitative sensation of "brightness" in the eye. Then from $\bar{y}(\lambda)$ we see that in order to produce the same sensation of brightness with red light of wavelength 650 nm requires 0.323/0.107 ≈ 3 times the radiance, i.e. $L(\lambda{=}650) \approx 3{\times}L(\lambda{=}500)$.

The sensitivity of the eye shifts toward the blue end of the spectrum in dim light; the *scotopic* luminosity function of Fig. 2.1 shows the relative response curve for this "night vision" case. The eye is most sensitive in bright light to a wavelength of 555 nm, and to a wavelength of 507 nm in dim light. Since most observation of natural waters takes place in daylight, we shall need only the photopic (bright light) response curve. However, any of the results developed below using the photopic luminosity function can be reformulated for the dim light case by simply replacing the photopic function by the scotopic function.

Table 2.1. The standard photopic luminosity function $\vec{x}$ and its integral, $I(\lambda) \equiv \int_{380}^{\lambda} \bar{y}(\lambda')\, d\lambda'$ (nm).

λ (nm)	$\bar{y}(\lambda)$	$I(\lambda)$ (nm)	λ (nm)	$\bar{y}(\lambda)$	$I(\lambda)$ (nm)
380	0.0000	0.000	580	0.8700	68.703
390	0.0001	0.000	590	0.7570	77.403
400	0.0004	0.001	600	0.6310	84.973
410	0.0012	0.005	610	0.5030	91.283
420	0.0040	0.017	620	0.3810	96.313
430	0.0116	0.057	630	0.2650	100.123
440	0.0230	0.173	640	0.1750	102.773
450	0.0380	0.403	650	0.1070	104.523
460	0.0600	0.783	660	0.0610	105.593
470	0.0910	1.383	670	0.0320	106.203
480	0.1390	2.293	680	0.0170	106.523
490	0.2080	3.683	690	0.0082	106.693
500	0.3230	5.763	700	0.0041	106.775
510	0.5030	8.993	710	0.0021	106.816
520	0.7100	14.023	720	0.0011	106.837
530	0.8620	21.123	730	0.0005	106.848
540	0.9540	29.743	740	0.0003	106.853
550	0.9950	39.283	750	0.0001	106.856
560	0.9950	49.233	760	0.0001	106.856
570	0.9520	59.183	770	0.0000	106.857

2.2 The Lumen

The photopic luminosity function gives the relative sensitivity of the human eye to different wavelengths. It remains to set an absolute magnitude to this sensation, and this is the task of the *lumen*, abbreviated "lm." To determine the luminous content of a sample of radiant power we need a conversion factor K_m, of units lumen per watt, which can convert *spectral radiant power* (in watts) into *luminous power* (in lumens) We therefore need some object of known and reproducible brightness as our absolute standard for luminosity. The object used is the surface of a hot platinum body at the temperature (2042 K) where it changes from liquid to solid at atmospheric pressure. By definition such a surface has a *luminance* (the photopically averaged radiance) of 6×10^5 *lumens* per square meter per steradian, that is

$$6\times10^5 \text{ lm m}^{-2} \text{ sr}^{-1} \equiv K_m \int_0^\infty L(\lambda)\,\bar{y}(\lambda)\,d\lambda\,. \tag{2.1}$$

Here $L(\lambda)$ is the spectral radiance of the melting platinum (in W m^{-2} sr^{-1} nm^{-1}). Notice that we have replaced watts by lumens in the transition from radiance to luminance. The seemingly arbitrary magnitude of 6×10^5 is tied by historical precedent to an earlier "standard candle" definition of the lumen. We can determine the radiance $L(\lambda)$ of melting platinum for each λ, because platinum at such a temperature and under controlled conditions takes on the appearance of a complete radiator (a black body). The celebrated Planck's law for blackbody radiation gives the spectral radiance $L_b(\lambda;T)$ of a complete radiator at wavelength λ (here in meters) and absolute temperature T (in Kelvins) as

$$L_b(\lambda;T) = \frac{c_1}{\lambda^5}\left[\exp\left(\frac{c_2}{\lambda T}\right) - 1\right]^{-1} \qquad (\text{W m}^{-2} \text{ sr}^{-1} \text{ m}^{-1})\,, \tag{2.2}$$

where $c_1 = 1.191\times10^{-16}$ W m^2 sr^{-1} and $c_2 = 1.439\times10^{-2}$ m K. Note that the wavelength interval in Eq. (2.2) is expressed in units of m^{-1}, since λ is in meters. Thus Eq. (2.1) becomes

$$6\times10^5 \text{ lm m}^{-2} \text{ sr}^{-1} = K_m \int_0^\infty L_b(\lambda; T = 2042\text{K})\,\bar{y}(\lambda)\,d\lambda\,. \tag{2.3}$$

This integral can be evaluated by use of Eq. (2.2); the result is

$$\int_0^\infty L_b(\lambda; 2042\,\text{K})\,\bar{y}(\lambda)\,d\lambda = 878 \text{ W m}^{-2} \text{ sr}^{-1}\,.$$

Solving Eq. (2.3) for K_m then yields

$$K_m = 683 \text{ lm W}^{-1}. \tag{2.4}$$

The conversion factor K_m is called the *maximum luminous efficacy.*

We note in passing that if the integral in Eq. (2.3) is evaluated with the scotopic luminosity function for $\bar{y}(\lambda)$, the resulting K_m is 1754 lm W^{-1}. Thus the dark-adapted eye is about two and one half times as efficient at converting radiant power into a visual sensation, as is the bright-light-adapted eye.

Table 1.1 shows the *candela* as the SI base unit of *luminous intensity.* By definition "the candela is the *luminous intensity*, in a given direction, of a source which is emitting monochromatic radiant energy of frequency 540×10^{12} Hz and whose *radiant intensity* in that direction is 1/683 W sr^{-1}." The lumen is then a derived quantity, which by definition is 1 lm ≡ 1 cd sr. These definitions are not particularly enlightening, and so for pedagogic reasons we have presented our discussion of photometry as though the lumen were the photometric base unit. Unsuccessful attempts have been made to have the lumen adopted as the SI base unit; the proposed definition being "the lumen is the luminous power of monochromatic radiant energy whose radiant power is 1/683 W and whose frequency is 540×10^{12} Hz." This frequency corresponds to $\lambda = 555$ nm for light in a vacuum.

2.3 Luminance

Having determined the conversion factor K_m, we can now give a precise definition of the capability of a given radiance distribution to evoke the physiological sensation of brightness. If $L(\vec{x};\hat{\xi};\lambda)$ is a given spectral radiance defined over the entire spectrum $0 \le \lambda < \infty$, then the corresponding luminance is denoted by $L_v(\vec{x};\hat{\xi})$ and is defined by

$$L_v(\vec{x};\hat{\xi}) \equiv K_m \int_0^\infty L(\vec{x};\hat{\xi};\lambda)\, \bar{y}(\lambda)\, d\lambda \qquad (\mathrm{lm\ m^{-2}\ sr^{-1}})\,. \tag{2.5}$$

Luminance is the quantity that most closely corresponds to the subjective concept of brightness.

There is no IAPSO recommended notation for photometric quantities. International standards organizations recommend using the same set of symbols for radiometirc and photometric quantities, but differentiating these quantities with a subscript *e* (for energy) for radiometric quantities and a subscript *v* (for visual) for photometric quantities, if there is a possibility of confusion. We shall include the *v* subscript on photometric quantities, but omit the *e* on radiometric quantities.

Examples

Consider a light source that generates a constant spectral radiance $L(\lambda) = L_o$ over the wavelength interval 300 nm $\le \lambda \le$ 800 nm, and which gives $L(\lambda) = 0$ outside this interval. Then the luminance of this source is

$$\begin{aligned} L_v &= K_m \int_0^\infty L(\lambda)\, \bar{y}(\lambda)\, d\lambda \\ &= K_m L_o \int_{300\ \mathrm{nm}}^{800\ \mathrm{nm}} \bar{y}(\lambda)\, d\lambda \\ &= 683 \times L_o \times 107 \;=\; 7.30\times10^4\, L_o \qquad (\mathrm{lm\ m^{-2}\ sr^{-1}})\,. \end{aligned}$$

As always, L_o has units of W m^{-2} sr^{-1} nm^{-1}. The value of the second integral was taken from the last line of column 6 in Table 2.1.

As another example of using Eq. (2.5), consider a light source that generates a radiance of $L = 1000$ W m^{-2} sr^{-1} nm^{-1} over an interval of $\Delta\lambda_1 = 10$ nm centered at $\lambda_1 = 450$ nm, a radiance of $L = 500$ W m^{-2} sr^{-1} nm^{-1} over an interval of $\Delta\lambda_2 = 5$ nm centered on $\lambda_2 = 600$ nm, and $L = 0$ otherwise. Then the luminance L_v of this source is

$$\begin{aligned} L_v &= K_m \int_0^\infty L(\lambda)\, \bar{y}(\lambda)\, d\lambda \\ &= K_m\, [L(450)\, \bar{y}(450)\, \Delta\lambda_1 \;+\; L(600)\, \bar{y}(600)\, \Delta\lambda_2] \\ &= 683\, [1000 \times 0.038 \times 10 \;+\; 500 \times 0.631 \times 5] \\ &= 1.34\times10^6\ \mathrm{lm\ m^{-2}\ sr^{-1}}\,. \end{aligned}$$

Here the values for $\bar{y}(450)$ and $\bar{y}(600)$ have been taken from Table 2.1.

Table 2.2 displays some typical luminances seen in nature.

Table 2.2. Typical luminances.

Source	Luminance ($\mathrm{lm\ m^{-2}\ sr^{-1} = cd\ m^{-2}}$)
solar disk, above the atmosphere	2×10^9
solar disk, at earth's surface, sun near the zenith	1×10^9
melting platinum	6×10^5
60 W frosted light bulb	1×10^5
sunlit snow surface	1×10^4
standard fluorescent light	8×10^3
full moon's disk	6×10^3
clear blue sky, directions away from the sun	3×10^3
heavy overcast, zenith direction	1×10^3
twilight sky	3
clear sky, moonlit night	3×10^{-2}
overcast sky, moonless night	3×10^{-5}

2.4 From Radiometry to Photometry

The transition from radiance to luminance, expressed by Eq. (2.5), can be repeated systematically for each radiometric quantity. We first define a general integral, the *radiometric-photometric transition operator,* by writing

$$\mathrm{Y}[\cdot] \equiv K_{\mathrm{m}} \int_0^\infty [\cdot]\, \bar{y}(\lambda)\, d\lambda\,. \tag{2.6}$$

In this notation, Eq. (2.5) reads

$$L_{\mathrm{v}}(\vec{x},\hat{\xi}) = \mathrm{Y}[L(\vec{x};\hat{\xi};\lambda)]\,.$$

The operator (2.6) is the general connection between radiometry and photometry. It permits all of the radiometric quantities developed in Chapter 1 to be carried over to the photometric context, without the necessity of any further geometrical argument.

Consider, for example, the effect of $\mathrm{Y}[\cdot]$ operating on the spectral downward plane irradiance $E_{\mathrm{d}}(\lambda)$

$$E_{\mathrm{dv}}(\vec{x}) \equiv \mathrm{Y}[E_{\mathrm{d}}(\vec{x};\lambda)] \tag{2.7a}$$

$$= K_{\mathrm{m}} \int_0^\infty E_{\mathrm{d}}(\vec{x};\lambda)\, \bar{y}(\lambda)\, d\lambda \tag{2.7b}$$

$$= K_{\mathrm{m}} \int_0^\infty \left[\int_{\hat{\xi} \in \Xi_{\mathrm{d}}} L(\vec{x};\hat{\xi};\lambda)\, |\cos\theta|\, d\Omega(\hat{\xi}) \right] \bar{y}(\lambda)\, d\lambda \tag{2.7c}$$

$$= \int_{\hat{\xi} \in \Xi_{\mathrm{d}}} \left[K_{\mathrm{m}} \int_0^\infty L(\vec{x};\hat{\xi};\lambda)\, \bar{y}(\lambda)\, d\lambda \right] |\cos\theta|\, d\Omega(\hat{\xi}) \tag{2.7d}$$

$$= \int_{\hat{\xi} \in \Xi_{\mathrm{d}}} L_{\mathrm{v}}(\vec{x};\hat{\xi})\, |\cos\theta|\, d\Omega(\hat{\xi}) \qquad (\mathrm{lm\ m^{-2}})\,. \tag{2.7e}$$

We have employed Eq. (2.6) in going from Eq. (2.7a) to (2.7b), recalled Eq. (1.23) in going to Eq. (2.7c), exchanged the order of integration in going to Eq. (2.7d), and used Eq. (2.5) in going to Eq. (2.7e). In Eq. (2.7a) we see how the downward *illuminance* E_{dv} (in lumens per square meter) is obtained from the spectral downward irradiance E_{d} (in watts per square meter per nanometer). Moreover, we see in Eq. (2.7e) that illuminance and luminance bear the same functional relation to each other as do irradiance and radiance.

Table 2.3. Summary of photometric concepts. Υ is the radiometric-photometric transition operator defined in Eq. (2.6). (Units can be converted to SI units by using 1 lm ≡ 1 cd sr.)

Concept	Units	Symbol	Definition
luminous energy	lm s	Q_v	$Q_v \equiv \Upsilon[Q]$
luminous power	lm = cd sr	Φ_v	$\Upsilon[\Phi]$
luminous intensity	lm sr^{-1} = cd	I_v	$\Upsilon[I]$
luminance	lm m^{-2} sr^{-1}	L_v	$\Upsilon[L]$
plane illuminance	lm m^{-2}	E_v	$\Upsilon[E]$
scalar illuminance	lm m^{-2}	E_{ov}	$\Upsilon[E_o]$
luminous exitance	lm m^{-2}	M_v	$\Upsilon[M]$

Table 2.3 summarizes the common photometric concepts on the same format as Table 1.5, which summarized the progenitor radiometric concepts. All other radiometric quantities, such as the vector irradiance, have their photometric counterparts, even if not shown in Table 2.3. The "field" and "surface" interpretations carry over into photometry. Thus the family of photometric concepts can be organized in a manner exactly analogous to the display of radiometric concepts in Fig. 1.8. Note finally that, owing to the linearity of the operator (2.6), any linear relation between radiometric quantities has a corresponding relation in the photometric setting. For example, the cosine law for irradiance becomes the cosine law for illuminance; the radiance invariance law becomes the luminance invariance law, and so on.

Older photometric literature contains a bewildering menagerie of units – footcandles, phots, stilbs, talbots, blondels, luxes, lamberts, and the like. Meyer-Arendt (1968) defines these non-SI units and presents handy tables of conversion factors.

2.5 Colorimetry

Colorimetry is that branch of science concerned with specifying *numerically* the color of a sample of radiant power $\Phi(\lambda)$ defined over the electromagnetic spectrum $0 \leq \lambda < \infty$. We now show how to compute and graphically display the color of an underwater scene or a color view of a natural water body seen from above.

For this purpose we adopt the widely used standard C.I.E. (*Commission Internationale de L'Eclairage*) color coordinate system, within which any sample of a spectral radiometric quantity can be located and assigned a unique color, in a manner to be explained below. By coupling the concepts of radiative transfer theory to the C.I.E. color coordinate system, an accurate, quantitative basis for the *description* and *prediction* of color phenomena is achieved.

The quantitative description of color

The C.I.E. color coordinate system is easily understood by analogy with vector analysis. Consider the resolution of a position vector $\vec{x}$ in a three-dimensional, cartesian coordinate system defined by $\hat{i}_1$, $\hat{i}_2$, and $\hat{i}_3$. The vector $\vec{x}$ has position components x_1, x_2, and x_3 given by

$$\begin{aligned} x_1 &= \vec{x}\cdot\hat{i}_1 \\ x_2 &= \vec{x}\cdot\hat{i}_2 \\ x_3 &= \vec{x}\cdot\hat{i}_3 , \end{aligned} \tag{2.8}$$

where $\vec{x}\cdot\hat{i}_1$ is the dot product between $\vec{x}$ and $\hat{i}_1$, etc. Finding (x_1,x_2,x_3) is known as the *analysis* of $\vec{x}$ in the given coordinate system. The *synthesis* of $\vec{x}$ refers to the recovery of $\vec{x}$ from its components:

$$\begin{aligned} \vec{x} &= x_1\hat{i}_1 + x_2\hat{i}_2 + x_3\hat{i}_3 \\ &= (\vec{x}\cdot\hat{i}_1)\,\hat{i}_1 + (\vec{x}\cdot\hat{i}_2)\,\hat{i}_2 + (\vec{x}\cdot\hat{i}_3)\,\hat{i}_3 . \end{aligned} \tag{2.9}$$

We can perform such an analysis and synthesis not only on a position vector $\vec{x}$, but also on any spectral radiometric function $F(\lambda)$ defined over the electromagnetic spectrum. Instead of the $\hat{i}_1$, $\hat{i}_2$, and $\hat{i}_3$ unit vectors, we now use the dimensionless *tristimulus functions* $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ as adopted by the C.I.E. in 1936. These tristimulus functions are shown in Fig. 2.2 and are tabulated in Table 2.4. The $\bar{x}(\lambda)$ and $\bar{z}(\lambda)$ functions are empirically determined, as is $\bar{y}(\lambda)$. The forms of these three functions result from the fact that the color receptors of the normal human eye (the cones) come in three distinct types, each of which is sensitive to only a part of the visible spectrum. Some of these receptors are most sensitive to red wavelengths, as reflected in $\bar{x}(\lambda)$. Those receptors most sensitive at green wavelengths determine $\bar{y}(\lambda)$, and the blue-sensitive receptors determine $\bar{z}(\lambda)$. It is this structure of the eye as a red-green-blue discriminator that, for example, enables a color television to produce the sensation of yellow in the viewer's eye-brain system by emitting roughly

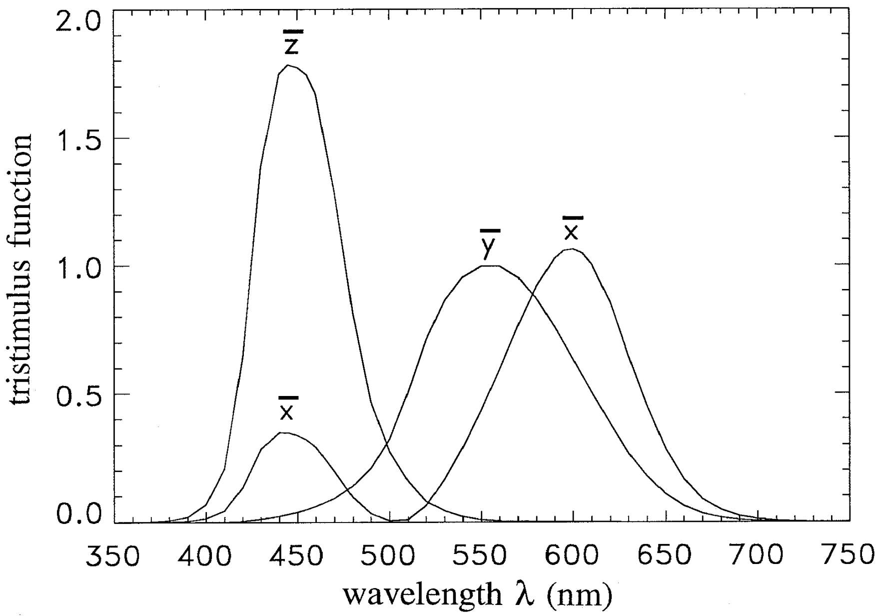

Fig. 2.2. The C.I.E. 1931 tristimulus functions.

equal parts of red and green (but no yellow) light. The green tristimulus function $\bar{y}(\lambda)$ is by definition identical to the photopic luminosity function of Section 2.1. This convenient choice for $\bar{y}(\lambda)$ results in $\bar{x}(\lambda)$ and $\bar{z}(\lambda)$ being somewhat different than the true spectral sensitivities of the associated cone cells.

The dot products of Eq. (2.8) are now replaced by the integrals

$$X \equiv K_{\mathrm{m}} \int_0^\infty F(\lambda)\, \bar{x}(\lambda)\, d\lambda\,, \tag{2.10a}$$

$$Y \equiv K_{\mathrm{m}} \int_0^\infty F(\lambda)\, \bar{y}(\lambda)\, d\lambda\,, \tag{2.10b}$$

$$Z \equiv K_{\mathrm{m}} \int_0^\infty F(\lambda)\, \bar{z}(\lambda)\, d\lambda\,. \tag{2.10c}$$

Here K_{m} = 638 lm W^{-1} is the maximum luminous efficacy determined in Section 2.2. Since $\bar{y}(\lambda)$ is the photopic luminosity function, Y defined in Eq. (2.10b) is the photometric quantity associated with the radiometric quantity $F(\lambda)$. If we take $F(\lambda)$ as the radiance $L(\lambda)$, then Y is just the luminance L_{v}, and so on.

Table 2.4. The CIE 1931 tristimulus, or color-matching, functions.[a]

λ (nm)	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$	λ (nm)	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$
360	0.0001	0.0000	0.0006	580	0.9163	0.8700	0.0017
370	0.0004	0.0000	0.0019	590	1.0263	0.7570	0.0011
380	0.0014	0.0000	0.0201	600	1.0622	0.6310	0.0008
390	0.0042	0.0001	0.0201	610	1.0026	0.5030	0.0003
400	0.0143	0.0004	0.0679	620	0.8544	0.3810	0.0002
410	0.0435	0.0012	0.2074	630	0.6424	0.2650	0.0001
420	0.1344	0.0040	0.6456	640	0.4479	0.1750	0.0000
430	0.2839	0.0116	1.3856	650	0.2835	0.1070	0.0000
440	0.3483	0.0230	1.7471	660	0.1649	0.0610	0.0000
450	0.3362	0.0380	1.7721	670	0.0874	0.0320	0.0000
460	0.2908	0.0600	1.6692	680	0.0467	0.0170	0.0000
470	0.1954	0.0910	1.2876	690	0.0227	0.0082	0.0000
480	0.0956	0.1390	0.8130	700	0.0114	0.0041	0.0000
490	0.0320	0.2080	0.4652	710	0.0058	0.0021	0.0000
500	0.0049	0.3230	0.2720	720	0.0028	0.0011	0.0000
510	0.0093	0.5030	0.1582	730	0.0014	0.0005	0.0000
520	0.0633	0.7100	0.0782	740	0.0007	0.0003	0.0000
530	0.1655	0.8620	0.0422	750	0.0003	0.0001	0.0000
540	0.2904	0.9540	0.0203	760	0.0002	0.0000	0.0000
550	0.4334	0.9950	0.0087	770	0.0001	0.0000	0.0000
560	0.5945	0.9950	0.0039	780	0.0000	0.0000	0.0000
570	0.7621	0.9520	0.0021	sum	10.684	10.686	10.679

a. Condensed from Wyszecki and Stiles (1982). Copyright 1982 by John Wiley & Sons, Inc. Reproduced by permission.

The integrals of Eq. (2.10) analyze a given spectral radiometric function $F(\lambda)$ into (X,Y,Z) = (red, green, blue) components. The corresponding synthesis formula gives the *color* or *chromaticity* of $F(\lambda)$:

$$\mathfrak{C}(F) \equiv X\bar{x}(\lambda) + Y\bar{y}(\lambda) + Z\bar{z}(\lambda) .$$

$\mathfrak{C}(F)$ is a function of wavelength designed to give a very close *visual color match* to the eye-brain sensation produced by the original $F(\lambda)$. The important point to note here is that although $F(\lambda)$ can be a completely arbitrary function of λ, its color $\mathfrak{C}(F)$ is the linear superposition of weighted amounts of standardized red, green and blue radiant power samples. The weights X, Y and Z are called the *color components* of F, and the ordered triple (X, Y, Z) is the associated *color vector.* Because the tristimulus functions are designed for matching the colors of radiant samples $F(\lambda)$, they are sometimes called the *color-matching* functions.

For a given light field in nature, different radiometric functions $F(\lambda)$ describing that light field in general have different color components. For example, if we look through a narrow tube pointed straight down at the sea surface, i.e. if we take the radiance $L(\hat{\xi};\lambda)$ with $\hat{\xi} = -\hat{i}_3$ as $F(\lambda)$, we may perceive a deeper blue than if we remove the tube and allow our eyes to receive photons traveling in any upward direction, in which case we are using an irradiance for $F(\lambda)$. Indeed, the 1931 C.I.E. tristimulus functions are designed for color matching when the field of view is only a few degrees. If the field of view is larger than 10°, slightly different tristimulus functions are recommended.

When working with position vectors $\vec{x}$, a special place is reserved for vectors of unit length, namely the unit sphere Ξ. In color specification, the *chromaticity plane,* represented by the largest triangle in Fig. 2.3, is the analogous construction. This plane has the property that for all points (x, y, z) lying on it, $x + y + z = 1$. If (X, Y, Z) is a color vector, then the vector

$$(x,y,z) \equiv \left(\frac{X}{X+Y+Z}, \frac{Y}{X+Y+Z}, \frac{Z}{X+Y+Z}\right) \tag{2.11}$$

lies on the chromaticity plane. We call x, y, and z the *chromaticity components* or *coordinates* of $F(\lambda)$. Since X, Y and Z are non-negative, only the part of the chromaticity plane lying in the first octant is needed in colorimetry. Observe also that since $x + y + z = 1$, only two of the three numbers x, y, z are needed to locate a point on the plane. By convention we use x and y. By projecting all color vectors (X, Y, Z) onto the chromaticity plane as illustrated in Fig. 2.3, we are in effect normalizing the photometric correspondent of $F(\lambda)$. For example, we are normalizing luminances if F is radiance. This normalization separates the attribute of brightness from that of color. Now that the chromaticity plane has been defined, we can simply project it down onto the x-y plane – that is, plot only the x and y values – in order to generate a convenient graphical display of color information.

Suppose we have a sample of radiance that is independent of λ over the region 360 nm $\le \lambda \le$ 780 nm [where $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are nonzero], and zero elsewhere. Then from Eq. (2.10),

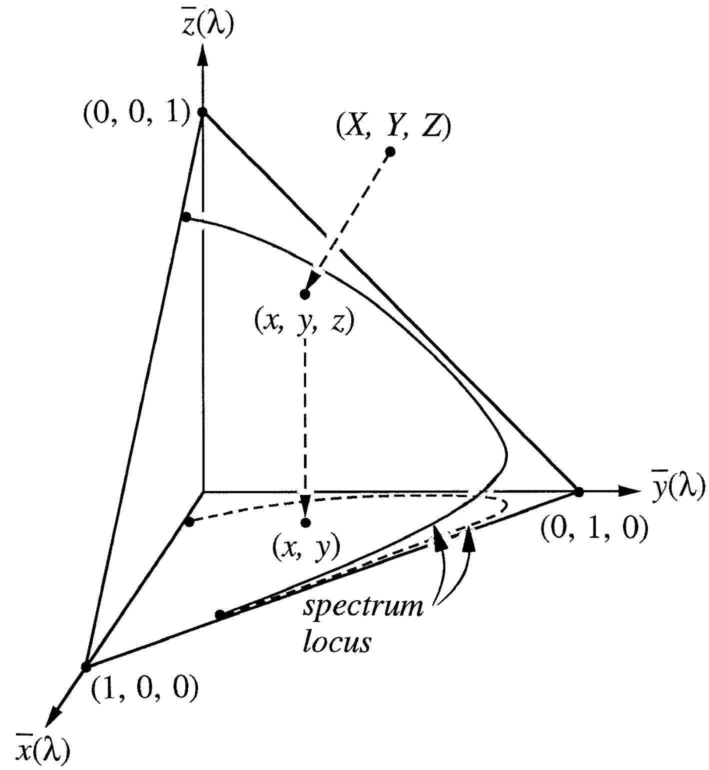

Fig. 2.3. The chromaticity plane, the spectrum locus, and its projection onto the $\bar{x}(\lambda)$ - $\bar{y}(\lambda)$ plane.

$$\begin{aligned} X &= K_{\mathrm{m}} \int_0^\infty L(\lambda)\,\bar{x}(\lambda)\,d\lambda = K_{\mathrm{m}} L \int_0^\infty \bar{x}(\lambda)\,d\lambda \\ &\approx K_{\mathrm{m}} L \sum_i \bar{x}(\lambda_i)\,\Delta\lambda_i = K_{\mathrm{m}} L\,\Delta\lambda \sum_i \bar{x}(\lambda_i) \\ &= 683 \times L \times 10 \times 10.684 \approx 7.3\times 10^4\,L\,. \end{aligned}$$

Here we have used the $\bar{x}(\lambda)$ values from Table 2.4, which are consistent with a value of $\Delta\lambda = 10$ nm. Likewise, $Y = Z = 7.3\times10^4\,L$. Then by Eq. (2.11) we get $x = y = z = 1/3$ as the chromaticity coordinates of this radiance sample. Wavelength-independent radiance has the visual appearance of pure white light. The *white-light point* $(x,y) = (1/3,1/3)$ is the central base of operations in the practical task of specifying colors.

If we next compute the chromaticities of all the pure monochromatic colors of the spectrum, we sweep out the horseshoe-shaped curve shown in Fig. 2.3. This curve, or its projection in the x-y plane, is called the *spectrum locus*. The x-y projection of the curve starts near the point (x,y) = (0.176, 0.005) (violet), sweeps around to (0.074, 0.834) (green), and ends up near (0.735, 0.265) (red). We close the spectrum locus by drawing a straight line, the *purple line*, between the red and violet points. The closed region so formed in the x-y plane is called a C.I.E. standard *chromaticity diagram.* Table 2.5 gives the chromaticity coordinates for the pure colors. [These coordinates can be found from the entries of Table 2.4 by summing across for each λ and dividing this sum into $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$.]

Table 2.5. Chromaticity coordinates for the spectrum locus (the pure colors).[a]

λ (nm)	x	y	z	λ (nm)	x	y	z
360	0.176	0.005	0.819	580	0.512	0.487	0.001
370	0.175	0.005	0.820	590	0.575	0.424	0.001
380	0.174	0.005	0.821	600	0.627	0.373	0.000
390	0.174	0.005	0.821	610	0.666	0.334	0.000
400	0.173	0.005	0.822	620	0.692	0.308	0.000
410	0.173	0.005	0.822	630	0.708	0.292	0.000
420	0.171	0.005	0.824	640	0.719	0.281	0.000
430	0.169	0.007	0.824	650	0.726	0.274	0.000
440	0.164	0.011	0.825	660	0.730	0.270	0.000
450	0.156	0.018	0.826	670	0.732	0.268	0.000
460	0.144	0.030	0.826	680	0.733	0.267	0.000
470	0.124	0.058	0.818	690	0.735	0.265	0.000
480	0.091	0.133	0.776	700	0.735	0.265	0.000
490	0.045	0.295	0.660	710	0.735	0.265	0.000
500	0.008	0.539	0.453	720	0.735	0.265	0.000
510	0.014	0.750	0.236	730	0.735	0.265	0.000
520	0.074	0.834	0.092	740	0.735	0.265	0.000
530	0.155	0.806	0.039	750	0.735	0.265	0.000
540	0.230	0.754	0.016	760	0.735	0.265	0.000
550	0.302	0.692	0.006	770	0.735	0.265	0.000
560	0.373	0.625	0.002	780	0.735	0.265	0.000
570	0.444	0.555	0.001				

a. Condensed from Wyszecki and Stiles (1982). Copyright 1982 by John Wiley & Sons, Inc. Reproduced by permission.

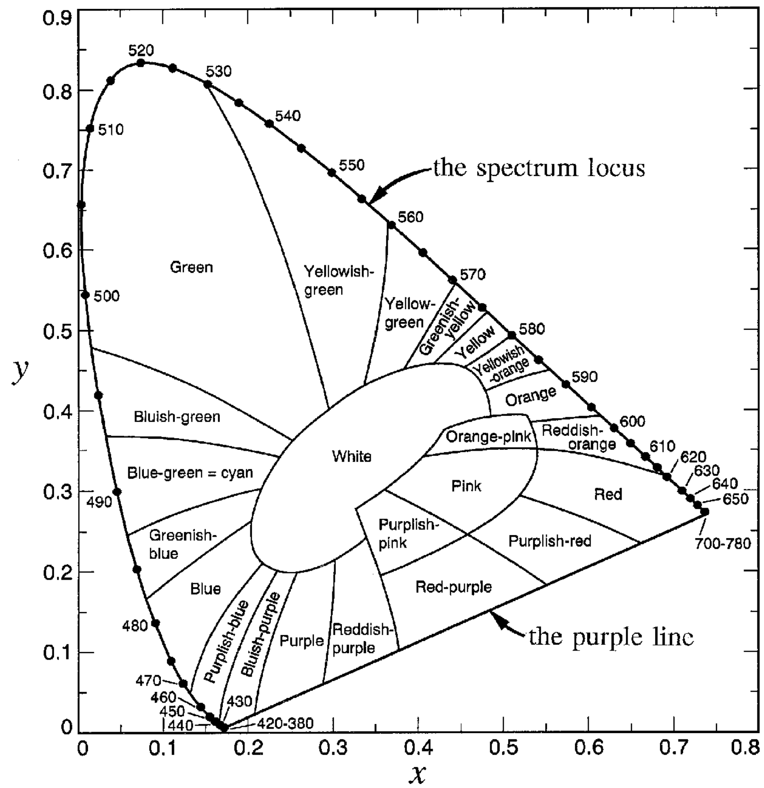

Fig. 2.4. A C.I.E. 1931 chromaticity diagram showing the regions associated with various colors. [redrawn from Kelly (1943), by permission]

Figure 2.4 shows a chromaticity diagram subdivided into regions of various colors. These divisions are rather crude, and a wide range of distinguishable colors is included within each division. Thus colors such as pink, rose, mauve, maroon, magenta and fuchsia may all fall within the pink region of Fig. 2.4. Likewise, a very "bright" yellow may lie in the white region of the diagram, although its (x,y) position will be on the yellow side of the pure white point at $(x,y) = (1/3,1/3)$.

Suppose a sample of radiant power $F(\lambda)$ has chromaticity coordinates (x,y) that land it at point P on the chromaticity diagram of Fig. 2.5. The white light point at $(x,y) = (1/3,1/3)$ is indicated by W. Now draw a straight line from W through P to intersect the spectrum locus at Q. The wavelength of the spectrum locus at Q is called the *dominant wavelength* λ_d of $F(\lambda)$. The fraction $p = WP/WQ$, where WP and WQ denote the lengths of the respective line segments, is defined as the *purity* of the color of $F(\lambda)$. The

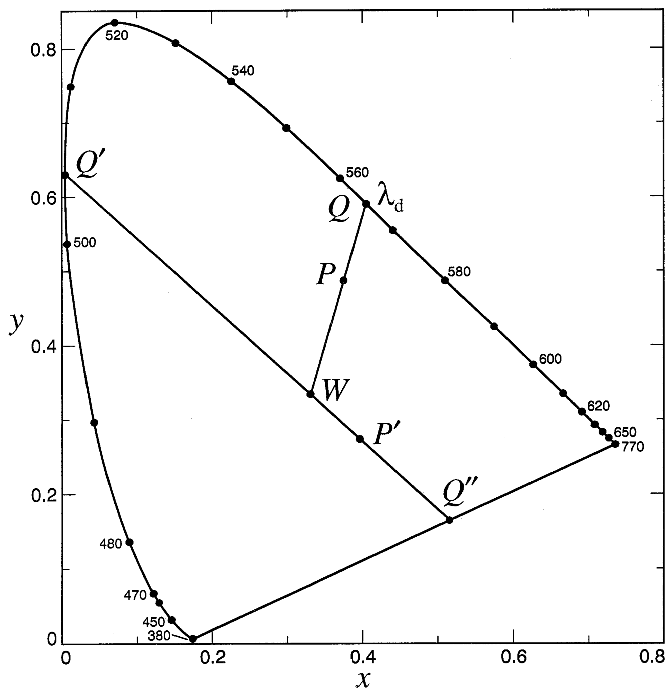

Fig. 2.5. Determination of dominant and complementary wavelengths, and of purity.

dominant wavelength is the wavelength of monochromatic light which, if mixed with pure white light in a proportion of p parts of monochromatic light and $1 - p$ parts of white light, will reproduce the color of the given $F(\lambda)$. Clearly, the pure (monochromatic) colors of the spectrum locus have a purity of $p = 1$. White light, the most "impure" mixture of all colors, has $p = 0$ and an undefined λ_d.

The color of points such as P' in the "purple region" of the chromaticity diagram cannot be reproduced by white light plus pure spectral light, and thus do not have a dominant wavelength. However, if we extend P' through W and on to Q' as before, the spectral wavelength at Q' is the *complementary* wavelength, and the associated purity is defined to be $p = WP'/WQ''$, as shown in Fig. 2.5. The complementary wavelength is the wavelength of pure spectral light that, if *removed* from white light in the proper proportion, will give a color match to $F(\lambda)$. That is to say, purple light is white light minus green light, which leaves a mixture of red and blue. Colors such as purple are sometimes called subtractive colors, for obvious reasons.

The standard reference work on photometry and colorimetry is the treatise by Wyszecki and Stiles (1982). Volumes I and II of *Hydrologic Optics* (1976) also contain a rigorous and general development of these subjects.

An example of experimentally determined chromaticity coordinates

Figure 2.6(a) depicts the spectral dependence of the radiance of submerged sandy shoals and reefs as observed by Duntley (1963) looking

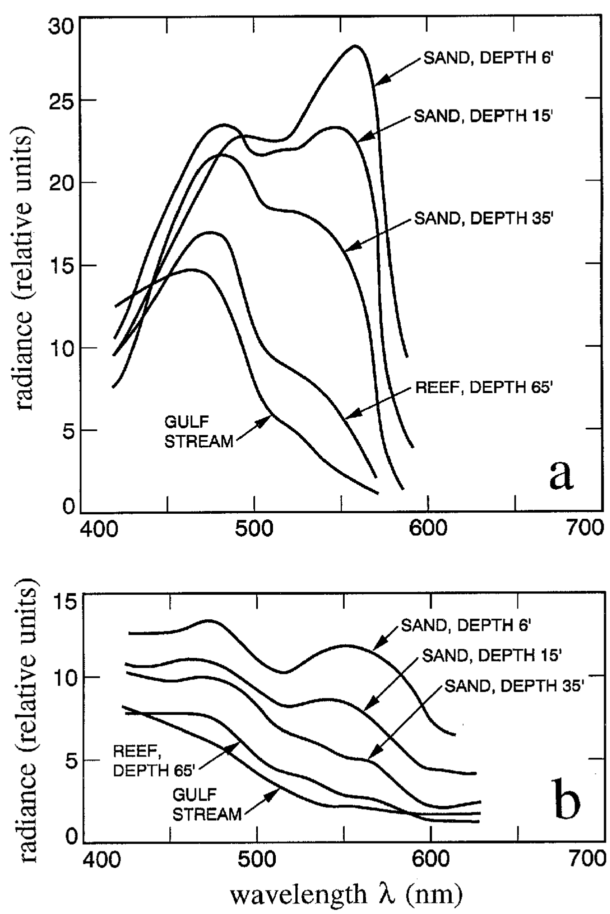

Fig. 2.6. Relative radiances for various bottom types as measured looking through the bottom of a glass-bottomed boat (panel a) and from an aircraft 1300 m above the sea surface (panel b). [redrawn from Duntley (1963), by permission]

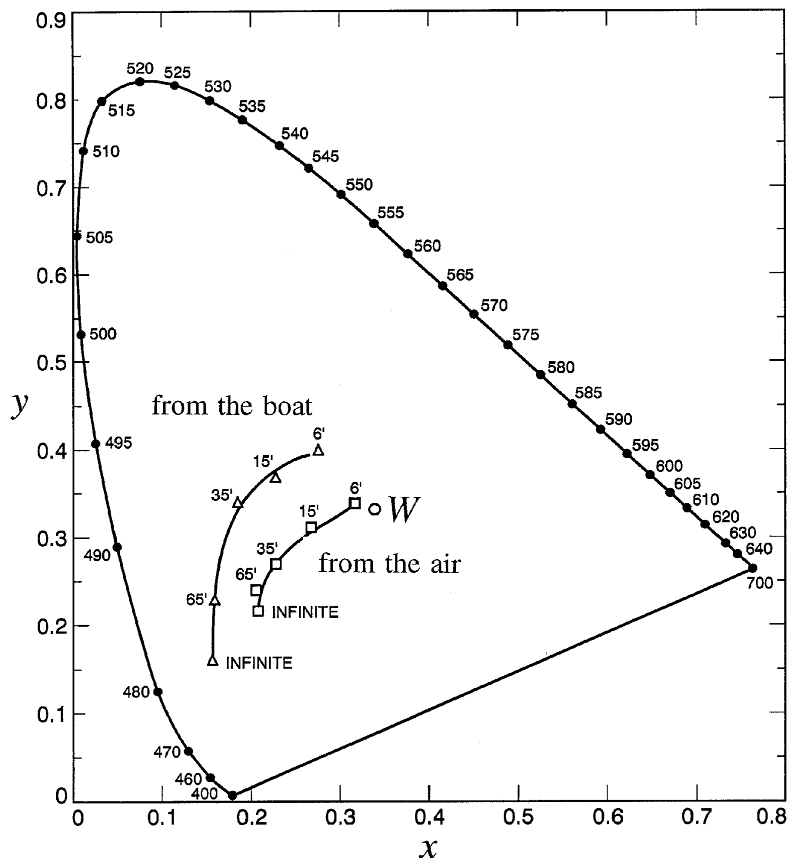

Fig. 2.7. Points on the chromaticity diagram as generated by the radiances of Fig. 2.6. The upper curve corresponds to Fig. 2.6(a), and the lower curve to Fig. 2.6(b). Point *W* is the white light point. [redrawn from Duntley (1963), by permission]

straight down through a glass-bottomed boat surveying parts of the east coast of Florida. Radiances for the same submarine area observed from an altitude of 4300 feet (1300 m) are depicted in Fig. 2.6(b). If $L(b;\lambda)$ is the spectral radiance of the underwater scene at a location where the bottom depth is b (feet), then the chromaticity coordinates for each bottom depth case are obtained by using the plotted radiances in Eqs. (2.10) and (2.11), with $L(b;\lambda)$ replacing $F(\lambda)$. The locations of the (x,y) chromaticity coordinates are plotted for the various bottom depths in the two curves of Fig. 2.7; the upper curve is for the radiances of Fig. 2.6(a) (from the boat) and the lower curve is for the radiances of Fig. 2.6(b) (from the air). Note on the upper curve that when the water is shallow [the point at $b = 6$ ft, or $(x,y) = (0.27,0.39)$], the color is an impure green with a dominant wavelength and purity of $(\lambda_d,p) = (507$ nm, $0.19)$. As the water depth increases, the dominant wavelength shifts toward the blue and the purity of the color increases. By the time the open ocean is reached (the "Gulf Stream" curve of Fig. 2.6, and the "infinite" depth point of Fig. 2.7), the color is a fairly pure, deep blue with $(\lambda_d,p) = (477$ nm, $0.76)$. Similar comments hold for the bottom-depth dependence of the water color as seen from the airplane; these data are plotted on the shorter curve of Fig. 2.7. Further examples of chromaticity diagrams for natural waters can be found in Jerlov (1976).

2.6 Problems

2.1. Figure 2.8 shows the relative spectral radiances of sunlight reflected from the covers of four important books: (a) *Marine Optics* by Jerlov (1976), (b) *Laser Remote Sensing* by Measures (1992), (c) *Absorption and Scattering of Light by Small Particles* by Bohren and Huffman (1983), and (d) *Ocean Optics XI*, edited by Gilbert (1992). For convenience in generating the figure, each measured $L(\lambda)$ was normalized to the largest value in the wavelength interval 350 nm to 750 nm. Compute the chromaticity coordinates of these book covers (as seen in sunlight). Values of $L(\lambda)$ read from the figure at 20 nm intervals should give a sufficiently accurate answer. What color would you call each book? You can check your answers by going to the library.

2.2. Suppose you have a light source that emits light with the same spectral distribution as the green tristimulus function, $\bar{y}(\lambda)$. What are the chromaticity coordinates (x,y) of this light? What are the dominant wavelength λ_d and purity p?

2.3. What are the chromaticity coordinates of an object that is radiating as a blackbody at a temperature of $T = 1000$ K? Could you describe the object as being "red hot"?

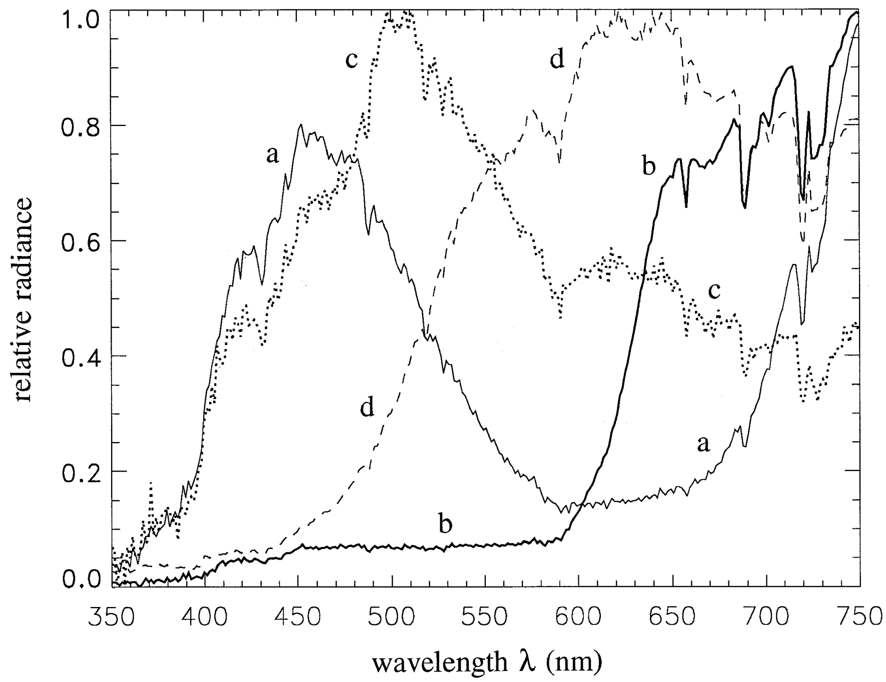

Fig. 2.8. Relative spectral radiances for use in problem 2.1.

Next Chapter >>