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PART I

FUNDAMENTALS

Part I of our book lays the physical and conceptual foundations of *hydrologic optics*, the quantitative study of the interactions of light with the earth's oceans, estuaries, lakes, rivers and other water bodies. *Radiative transfer theory* is the physical and mathematical framework for hydrologic optics.

Chapter 1 begins with a few comments upon the nature of light itself and upon the sun, earth's primary source of light. We then briefly describe a few of the instruments used to quantitatively measure the flow of electromagnetic energy. Various concepts used in classifying electromagnetic energy are defined and discussed in some detail. These concepts – power, radiance, irradiance, and intensity – lie within the purview of geometrical radiometry and provide a complete set of building blocks for the mathematical structure of radiative transfer theory.

Chapter 2 surveys the subject of photometry, the study of how radiant energy is perceived by the human eye and brain. Strictly speaking, this material is extraneous to the study of radiative transfer. Yet it is photometry, not the physical science of radiometry, that enables us to understand the connections between radiant energy and subjective concepts such as brightness and color.

In Chapters 1 and 2 we learn how to measure and describe light; in Chapter 3 we learn how to specify the optical properties of the medium through which the light propagates. Inherent and apparent optical properties are defined, and we review the current literature on values and bio-optical models for the absorption, scattering, and attenuation properties of natural waters.

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Chapter 1

Radiometry

Radiometry is the science of the measurement of electromagnetic, or radiant, energy. It forms the cornerstone of radiative transfer studies in natural waters. In this chapter we introduce the fundamental concepts of radiometry and discuss, in particular, geometrical radiometry.

We adopt the International System of Units, commonly called SI (for Système International) units. This system rests on seven base units and two supplementary units, as shown in Table 1.1. All other quantities are derivable from these units. Several derived units that we shall presently need are shown in Table 1.2. Detailed definitions of the SI base and derived units are found in NBS Special Publication 330 (1981).

In our choice of nomenclature and symbols we generally follow the recommendations of the Committee on Radiant Energy in the Sea of the International Association of Physical Sciences of the Ocean (IAPSO; see Morel and Smith, 1982). This is the nomenclature most widely used today by researchers in hydrologic optics, and we adopt it out of a desire to have a common language for our work. However, the recommended notation is not entirely satisfactory. In particular, it is incomplete and is often

Table 1.1. SI base units.

Physical quantity	Base unit	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd
	supplementary units	
plane angle	radian	rad
solid angle	steradian	sr

Table 1.2. Derived units useful in radiative transfer studies.

Physical quantity	Derived unit	Symbol	Definition
wavelength of light	nanometer	nm	10^{-9} m
energy	joule	J	1 kg m^2 s^{-2}
power	watt	W	1 kg m^2 s^{-3}
number of photons	einstein	einst	1 mol of photons (6.023×10^{23} photons)

inconvenient for mathematical manipulations; consequently we occasionally shall make minor deviations from the IAPSO recommendations.

1.1 The Nature of Light

Light is reasonably well envisioned as consisting of numerous localized packets of electromagnetic energy, called *photons*, which move through empty space with speed $c = 2.998\times10^8$ m s^{-1}. Each of these "particles" of light also carries with it a certain linear momentum and angular momentum. But the photon is not a particle in the sense of classical mechanics, for associated with it are a wavelength λ and a frequency ν, which endow the photon with wave-like properties.

Ascribing both wave and particle properties to a photon is an instance of the wave-particle duality of all matter and energy; this duality is a well-established cornerstone of modern physics. Thus a photon is not *either* a particle *or* a wave; both aspects are necessary for a proper understanding of light. Indeed, if we perform an experiment that is designed to detect the wave nature of light (e.g. one using diffraction gratings), then we do detect the wave properties (e.g. we can measure the wavelength). On the other hand, if we perform an experiment that is designed to detect the particle nature of light (e.g. one involving absorption, as in the photoelectric effect), then we observe only the particle properties (e.g. light energy comes in discrete units).

We also may envision a photon as a localized region of time-varying, self-sustaining electric and magnetic fields. According to Maxwell's equations, a time-dependent electric field generates a time-dependent magnetic field, and conversely. The frequency of these oscillating electric and magnetic fields is the photon's frequency ν. The electromagnetic field description of light is useful for many applications. It is, for example, the basis of Mie scattering theory, which will be discussed later. However, the photon "particle" viewpoint is adequate for our discussion of radiative transfer theory, and we need not delve further into electromagnetic theory.

However, no experiment has ever been devised that indicates that the photon has any internal structure; photons (perhaps like electrons) truly may be mathematical points, without physical size. Note that if a photon has a wavelength of λ = 1 m, for example, this does not mean that the photon "particle" is one meter long; it merely means that the photon has an "internal" frequency ν that corresponds to $\lambda \equiv c/\nu$ = 1 m. These topics may be pursued in textbooks on modern physics (e.g. Eisberg and Resnick, 1985). A fascinating, non-mathematical exposition of our current understanding of light, and of light's role in the fundamental structure of nature, is found in Feynman (1985).

The photon viewpoint of light is well suited to our development of radiative transfer theory. However, the electromagnetic-field viewpoint is more convenient for certain types of problems, as we shall note at the appropriate times.

The energy q of a photon is related to its frequency ν and corresponding wavelength λ by

$$q = h\nu = \frac{hc}{\lambda}, \tag{1.1}$$

where $h = 6.626\times10^{-34}$ J s is Planck's constant. The *magnitudes* of the linear momentum p and angular momentum l of a photon are given by

$$p = \frac{h}{\lambda} \tag{1.2}$$

and

$$l = \frac{h}{2\pi}, \tag{1.3}$$

respectively.

For example, a photon of wavelength λ = 550 nm = 550×10^{-9} m (green light) carries radiant energy

$$q = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \mathrm{J\ s})\,(2.998\times10^{8}\ \mathrm{m\ s^{-1}})}{550\times10^{-9}\ \mathrm{m}} = 3.61\times10^{-19}\ \mathrm{J},$$

linear momentum of magnitude

$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ \mathrm{J\ s}}{550\times10^{-9}\ \mathrm{m}} = 1.20\times10^{-27}\ \mathrm{kg\ m\ s^{-1}},$$

and angular momentum of magnitude

$$l = \frac{h}{2\pi} = \frac{6.626\times10^{-34}\ \mathrm{J\ s}}{6.283} = 1.05\times10^{-34}\ \mathrm{J\ s}\,.$$

Note that the energy and linear momentum both depend on the wavelength, but that all photons have the same angular momentum. In addition, every photon has a definite *state of polarization*, which is related to the direction of the plane of vibration of the photon's electric field.

1.2 Light from the Sun

In this section we briefly survey how light is produced by the sun, and we characterize the sunlight reaching the earth.

Solar energy production

Since most of the light falling upon the earth originates in the sun, it is worthwhile to take a brief look at how this light is generated deep within the sun in a process known as the proton-proton cycle.

The core of the sun is primarily a mixture of completely ionized hydrogen and helium. At the center the temperature is approximately 15×10^6 K, and the density is about 150 times that of water. Under these extreme conditions the hydrogen nuclei, or protons, occasionally collide with sufficient energy to overcome their electrostatic repulsion and fuse together according to the reaction

$$\text{proton} + \text{proton} \rightarrow \underbrace{(\text{proton} + \text{neutron})}_{\text{deuteron}} + \text{positron} + \text{neutrino}. \tag{1.4}$$

This reaction releases energy, which is carried off by the created particles. The bound state of a proton and a neutron is called a deuteron. The positron is identical to an electron except for its electric charge, which is positive. The neutrino is an uncharged, massless (or nearly massless) packet of energy that is in some ways similar to a photon.

The deuterons are able to undergo further fusion with protons to form an isotope of helium:

$$\text{deuteron} + \text{proton} \rightarrow \underbrace{(2\ \text{protons} + \text{neutron})}_{\text{Helium}^3} + \text{photon}. \tag{1.5}$$

This reaction also creates a photon, or particle of light, which in fact carries off most of the energy released in the reaction. $Helium^3$ nuclei are in turn able to fuse with each other in the final step of the proton-proton cycle:

$$\text{Helium}^3 + \text{Helium}^3 \rightarrow \underbrace{(2 \text{ protons } + 2 \text{ neutrons})}_{\text{Helium}^4} + 2 \text{ protons.} \tag{1.6}$$

In order to create the two $Helium^3$ nuclei of step (1.6), there must be two each of reactions (1.4) and (1.5). Thus if we tally the total input, we find that six protons have been converted to a total output of one $Helium^4$ nucleus, two protons, two positrons, two neutrinos and two photons. The positrons each soon encounter electrons and undergo mutual annihilation to create two or more photons:

$$\text{positron} + \text{electron} \rightarrow \text{photons.}$$

Therefore the net result of the proton-proton cycle is the conversion of four hydrogen atoms into one helium atom plus energy in the form of photons and neutrinos. The energy released in these reactions is tremendous: about 6.4×10^{14} J per kilogram of hydrogen converted.

Neutrinos interact only weakly with matter, and they escape from the sun immediately after their creation in step (1.4). The various photons created in the above reactions are all extremely energetic (in the gamma ray region of the electromagnetic spectrum). They also interact strongly with matter, and therefore the gamma rays undergo repeated scattering, absorption and re-emission by the solar matter as they work their way toward the surface of the sun. The photons lose energy in these interactions, so that they are predominantly in the visible and infrared parts of the spectrum by the time they arrive at the sun's surface and escape into space.

There are exothermic nuclear reactions other than the ones just described; details may be found in books on stellar evolution (e.g. Clayton, 1983). However, the proton-proton cycle is responsible for almost all of the energy generation within our own sun.

Solar energy arriving at earth

The photons generated by the sun stream into space in all directions away from the sun. By conservation of energy, the total energy per unit time crossing an imaginary spherical surface of radius R measured from the sun's center is independent of R. However, since the area $4\pi R^2$ of the spherical surface increases as R^2, the energy per unit time *per unit area of the sphere*, which is called the *irradiance*, must decrease as R^{-2}. This simple result is known as the *inverse square law for irradiance.*

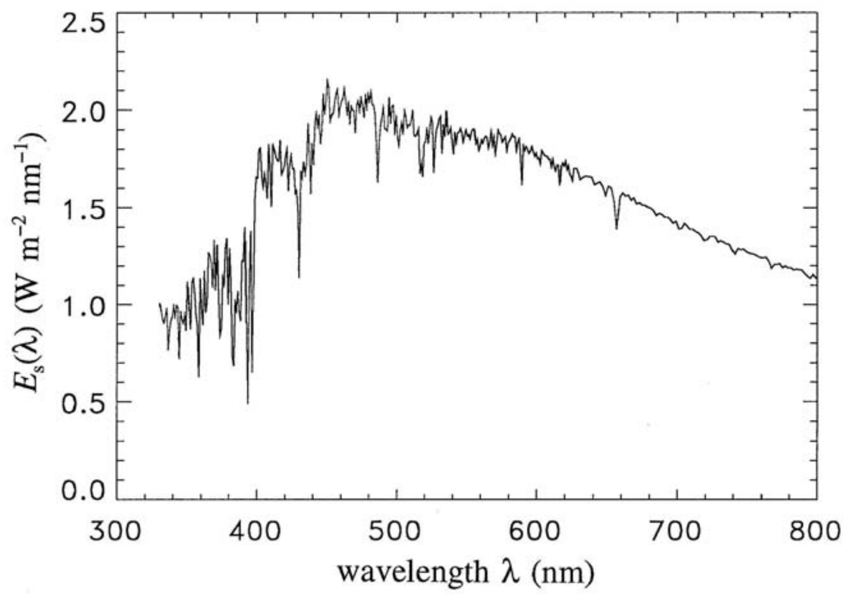

Fig. 1.1. Measured solar spectral irradiance at the mean earth-sun distance. The resolution is 1 nm for $\lambda \leq 630$ nm, and 2 nm for $\lambda > 630$ nm. [plotted from data tabulated in Neckel and Labs (1984)]

At the mean distance of the earth from the sun, the solar irradiance from photons of all wavelengths, E_s, is near (Frölich, 1983)

$$E_s = 1367 \text{ W m}^{-2}.$$

Although E_s is commonly called the *solar constant*, its value varies by a fraction of a percent on time scales of minutes to decades (Frölich, 1987). Moreover, the solar irradiance received by the earth varies about E_s by almost ± 50 W m^{-2} over the course of a year, owing to the ellipticity of the earth's orbit about the sun.

The photons arriving at earth from the sun are not all equally energetic. As we saw in Eq. (1.1), the energy of a photon is inversely proportional to its wavelength. Furthermore, the number of photons per wavelength interval is not uniform over the electromagnetic spectrum. Figure 1.1 shows the measured *spectral,* or wavelength, distribution of the solar irradiance*,* $E_s(\lambda)$, over the wavelength band of oceanographic interest. The sharp dips in the $E_s(\lambda)$ curve are *Fraunhofer lines*, which are due to selective absorption of solar radiation by elements in the sun's outer atmosphere. These lines are typically less than 0.1 nm wide, and they are much "deeper" (the irradiances within the lines are much less) than indicated in Fig. 1.1, which gives $E_s(\lambda)$ values averaged over bands 1 or 2 nm wide. For example, the prominent line centered at $\lambda = 486.13$ nm decreases to 0.2 of the E_s values just outside the line at $\lambda =$ 486.05 and 486.25; the *line depth* is thus said to be 0.2. We shall mention an interesting use of Fraunhofer lines at the end of Section 10.4.

However, it is seldom necessary for optical oceanographers and limnologists to concern themselves with the detailed wavelength dependence of $E_s(\lambda)$ seen in Fig. 1.1. It is usually sufficient to deal with $E_s(\lambda)$ values averaged over bandwidths of order $\Delta\lambda \approx 10$ nm, which correspond to the bandwidths of the optical instruments routinely used in underwater measurements. Table 1.3 gives the distribution of E_s in several broad wavelength bands. Note that only about 42% of the sun's energy is in the near-ultraviolet and visible bands relevant to hydrologic optics.

Moreover, it is not the solar irradiance at the top of the atmosphere, but rather the sunlight that actually reaches the sea surface, that is relevant to hydrologic optics. The magnitude and spectral dependence of the solar radiation reaching the earth's surface are highly variable functions of the solar angle from the zenith (i.e. of the time of day, season and latitude) and of atmospheric conditions (e.g. of humidity, dustiness , and cloud cover). Figure 1.2 illustrates how much of the direct, or unscattered, solar beam reaches the sea surface, as predicted by the LOWTRAN 7 atmospheric radiative transfer model (Kneizys, *et al.*, 1988). In that figure, curve *a* is

Table 1.3. Distribution of the solar constant in various wavelength bands.

Band	Wavelength interval (nm)	Irradiance ($W\,m^{-2}$)	Fraction of E_s (percent)[a]
ultraviolet and beyond	< 350	62	4.5
near ultraviolet	350-400	57	4.2
visible	400-700	522	38.2
near infrared	700-1000	309	22.6
infrared and beyond	> 1000	417	30.5
totals		1367	100.0

a. Percentages computed from data in Thekaekara (1976)

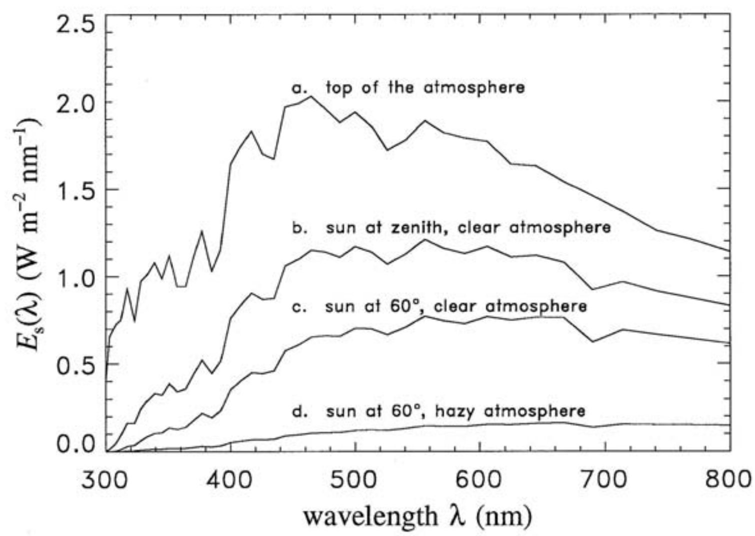

Fig. 1.2. Spectral irradiance of the sun's direct beam averaged over wavenumber intervals of size 500 cm^{-1}, as computed by LOWTRAN 7 (Kneizys, *et al*., 1988). Curve *a* gives $E_s(\lambda)$ at the top of the atmosphere; curves *b-d* give $E_s(\lambda)$ at sea level for various solar zenith angles and atmospheric conditions.

$E_s(\lambda)$ at the top of the atmosphere, averaged over various wavelength bands[1] $\Delta\lambda$. Curve *b* gives the solar direct beam irradiance at the sea surface when the sun is at the zenith, for a clear marine atmosphere (horizontal visibility of 23 km). Curve *c* is for the same atmosphere but with the sun at a zenith angle of 60° (i.e. 30° above the horizon; $E_s(\lambda)$ is measured on a surface perpendicular to the sun's rays). Curve *d* is for the sun at a zenith angle of 60°, but for a very hazy marine atmosphere (5 km visibility).

In addition to the direct beam irradiance seen in Fig. 1.2, comparable amounts of diffuse irradiance, consisting of photons that have been scattered out of the sun's direct beam by the atmosphere, reach the earth's surface (Tran and Collins, 1990). Table 1.4 gives typical values of the diffuse plus

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[1]A constant wavenumber interval of $\Delta\kappa$ = 500 cm^{-1} was used for averaging. This $\Delta\kappa$ corresponds to $\Delta\lambda$ values of 4.5 to 32 nm over the 300-800 nm band; see Eq. (1.32).

Table 1.4. Typical total (direct plus diffuse) irradiances at sea level in the visible wavelength band (400-700 nm).

Environment	Irradiance ($W\ m^{-2}$)
solar constant (for comparison)	522
clear atmosphere, sun near the zenith	500
clear atmosphere, sun at 60° from the zenith	450
hazy atmosphere, sun at 60° from the zenith	300
hazy atmosphere, sun near the horizon	100
heavy overcast, sun near the horizon	10
clear atmosphere, full moon near the zenith	1×10^{-3}
clear atmosphere, starlight only	3×10^{-6}
cloudy night	3×10^{-7}
clear atmosphere, light from a single, bright (1st magnitude) star	3×10^{-9}
clear atmosphere, light from a single, barely visible (6th magnitude) star	3×10^{-11}

direct visible irradiances reaching sea level for various environmental conditions.

Table 1.4 shows that several hundred watts per square meter of visible light are incident on the ocean's surface for typical daytime conditions. The first law of thermodynamics (see, for example, Halliday and Resnick, 1988) can be used to relate the rate of energy absorption by a body to the rate of change of its temperature:

$$\frac{\partial T}{\partial t} = \frac{1}{c_v m}\frac{\partial Q}{\partial t}\,.$$

Here T is temperature, t is time, c_v is the specific heat capacity of the body (c_v = 3900 J kg^{-1} K^{-1} for seawater), m is the mass of the body, and $\partial Q/\partial t$ is the rate of energy absorption. If we assume that five percent of the incident visible irradiance is absorbed within the upper one meter of the ocean, then for a typical irradiance of 400 W m^{-2} (i.e. Q = 400 J on each square meter each second) we find

$$\frac{\partial T}{\partial t} = \frac{1}{(3900\ \mathrm{J\ kg^{-1}\ K^{-1}})\,(10^3\ \mathrm{kg})}\left(\frac{0.05\times400\ \mathrm{J}}{1\ \mathrm{s}}\right) \approx 5\times10^{-6}\ \mathrm{K\ s^{-1}}\,.$$

In one ten-hour day, this heating rate would increase the water temperature of the ocean's upper meter by

$$\Delta T \approx (5\times10^{-6}\ \mathrm{K\ s^{-1}})\,(3.6\times10^{4}\ \mathrm{s}) \approx 0.2\ \mathrm{K}\,,$$

which is a very significant change in water temperature from the viewpoint of physical oceanography. Including the contribution by non-visible radiation, most of which is absorbed very near the surface, considerably increases the heating over that calculated here for visible light.

The uneven heating of the world's land and water surfaces is of course the fundamental process that drives the global circulation of the oceans and atmosphere. Sunlight also provides the energy for photosynthesis, the fundamental biological process that supports life on earth.

It is also instructive to estimate how much momentum sunlight transports to the oceans. The number N of photons falling on each square meter of ocean surface can be estimated from Eq. (1.1). Taking once again a typical irradiance of 400 W m^{-2} (i.e. Q = 400 J), and assuming that the "average" photon has a wavelength of λ = 550 nm, gives

$$N = \frac{Q}{q} = \frac{Q\,\lambda}{h\,c} = \frac{(400\ \mathrm{J})\,(550\times10^{-9}\ \mathrm{m})}{(6.63\times10^{-34}\ \mathrm{J\ s})\,(3.00\times10^{8}\ \mathrm{m\ s^{-1}})} \approx 10^{21}\,.$$

The magnitude of the linear momentum transported each second to a square meter of ocean surface by these photons is, by Eq. (1.2),

$$p = N\frac{h}{\lambda} = \frac{(10^{21})\,(6.63\times10^{-34}\ \mathrm{J\ s})}{(550\times10^{-9}\ \mathrm{m})} \approx 10^{-6}\ \mathrm{kg\ m\ s^{-1}}\,.$$

This is comparable to the linear momentum of only one cubic millimeter of water moving in a current of speed 1 m s^{-1}. Thus the linear momentum transported to the oceans by incident sunlight is completely negligible compared to that transported by wind and other processes that drive currents.

Similarly, Eq (1.3) gives the maximum possible angular momentum carried by these photons:

$$l = N\frac{h}{2\pi} = (10^{21})(1.05\times10^{-34}\ \mathrm{J\ s}) \approx 10^{-13}\ \mathrm{kg\ m^{2}\ s^{-1}}\,,$$

which is comparable to the angular momentum of a small sand grain rotating at one revolution per second. This angular momentum is much less than that of even the smallest turbulent eddies generated by current shears or breaking waves. Moreover, the maximum angular momentum as computed here is transferred only if all photons are in the same angular momentum state; i.e. only if the light is circularly polarized. For unpolarized sunlight, the net transfer of angular momentum is zero (Hecht, 1987).

Thus we see that *the physical importance of the sunlight incident on the ocean surface lies in its energy transport*, and not in its momentum transport. This is consistent with our intuition: sunlight heats us up rather quickly, but it does not push us around.

This introductory discussion of solar radiation is not intended to imply that artificial light sources (such as lasers) or other natural light sources (such as bioluminescence) are unimportant in hydrologic optics studies. However, our attention in this book is directed to those situations in which the light incident on the water surface is horizontally uniform over the region of interest. Most of the numerical techniques to be developed in the course of our discussions are built on the assumption of lateral homogeneity. Sunlight generally satisfies this assumption, whereas other lights are often better approximated as point sources (or even more complicated distributions of sources), in which case a somewhat different mathematical treatment of radiative transfer is required. Moreover, the primary concerns of oceanographers and limnologists – heating of water bodies, phytoplankton productivity, passive remote sensing, and the like – are well addressed if we consider only solar radiation.

1.3 Measuring Radiant Energy

The normal human eye is a sensitive detector of radiant energy, and it has an extremely wide *dynamic range* (the ratio of maximum to minimum detectable signal). However, the precise work of hydrologic optics requires a more objective means of measuring the flow of radiant energy. Two main classes of light detectors have been developed to detect and measure radiant energy: *thermal* and *quantum* detectors. In thermal detectors, radiant energy is absorbed and converted into heat energy, and the detector responds to the consequent change in temperature of the absorbing medium. Thermal detectors include ordinary thermometers, thermocouples, bolometers, and pyranometers. Quantum detectors respond directly to the number of incident photons, rather than to the cumulative energy carried by the photons, although the photon energy generally affects detector performance. Quantum detectors include photographic film and various photovoltaic, photoconductive, and photoemissive detectors. A brief description of the latter three detectors, collectively called *photoelectric* devices, is worthwhile.

A *photovoltaic* cell consists of two dissimilar substances in contact. Light incident on the photovoltaic element generates a difference of electric potential between the two dissimilar parts of the element, and as a consequence a current flows in the electrical circuit containing the cell. This current is measured by a current meter included in the circuit. When no light is incident on the element, no potential difference is generated and consequently no current flows. Generally, the greater the number of photons incident on the element of the cell, the greater is the resultant potential, and the greater is the ensuing current in the circuit. Becquerel first observed the photovoltaic effect in 1839 when a liquid electrolyte containing two immersed electrodes connected through a galvanometer was irradiated by sunlight.

It was found experimentally in 1873 that the electrical conductivity of the metal selenium increases when light falls upon it. This effect can be exploited for measuring radiant energy by constructing a series electrical circuit consisting of the selenium (or a similarly behaving substance), a seat of electromotive force (e.g. a battery), and a current meter. The greater the number of photons falling on the *photoconductive cell* containing the selenium, the greater is the cell's conductivity, and hence the greater is the current flowing in the circuit. Some *dark current* flows even if no light is incident on the cell, since the photoconductive substance has a nonzero conductivity even in the absence of light.

The photoemissive effect (often called the photoelectric effect) was discovered in crude form in 1887 by Hertz in the very same experiment in which he verified the existence of electromagnetic waves. The basic *photoemissive cell* consists of an evacuated tube containing a negatively charged electrode (the photocathode, usually made of an alkali metal such as cesium, sodium, or potassium) and a positively charged electrode (the anode). When light is incident on the photocathode, the photons dislodge electrons from the surface of the electrode. These *photoelectrons* are drawn across a gap to the anode, thus generating a current in a series circuit containing the cell, a current meter, and a seat of electromotive force, which replenishes the supply of electrons on the photocathode and maintains the potential difference across the electrodes. In principle, no current would flow if no light were incident on the photocathode, but in practice a small dark current flows because of electrons spontaneously emitted by random thermal motions in the cathode.

A *photomultiplier tube* (PMT) is a specialized photoemissive cell. Rather than having only one photocathode and one anode, a PMT has a series of anodes (called dynodes), each of which is held at higher positive voltage that the previous one. The electrons liberated from the photocathode by the incident light are attracted to the first dynode. When these original electrons strike the first dynode, they knock loose additional electrons, which are then attracted to the second dynode. The electrons striking the second dynode liberate still more electrons, which are attracted to the third dynode, and so on. This *electron cascade* enables a PMT to greatly amplify (typically by a factor of one million) the current which would result from the photoelectrons alone. Commercially available PMT's have up to 15 dynodes and are extremely sensitive light detectors. However, PMT response is very sensitive to temperature, the response is not stable with time (owing to changes in the dynodes caused by electron bombardment), and stable high-voltage power supplies are required for operation. For these and other reasons, PMT's have been supplanted in many oceanographic instruments by solid-state detectors.

Semiconductor diodes can serve as light detectors, in which case they are called *photodiodes*. For example, in a typical pn-junction silicon photodiode, light incident on the junction frees electrons from the silicon atoms (but does not eject the electrons from the diode). The resulting positively charged silicon ions are held fixed in position by the crystal lattice, whereas the free electrons can move in response to an applied electromotive force. These electrons thus generate a current when the photodiode is included on a series circuit with a seat of electromotive force and a current meter. The diode thus functions as a photoconductive cell. Note that a photodiode does not amplify the photocurrent as does a PMT, and consequently photodiodes are much less sensitive detectors than are PMT's. However, photodiodes have good stability, are easy to calibrate, require little power, and are quite rugged and inexpensive.

When operated as light detectors, diode junctions have the external electromotive force applied so as to separate the photoelectrons and their parent ions, thus generating the measured current. However, if the electrons are allowed to recombine with the ions, then photons are emitted from the junction. These photons have the same energy as the photons required to liberate electrons from the semiconductor atoms. When operated in this fashion, the diode is called a *light-emitting diode* (LED). LED's have the same general characteristics (stability, low cost, etc.) as photodiodes, and are often employed as light sources in oceanographic instruments (such as beam transmissometers) that require an internal light source.

Another type of photoelectric detector is the *charge-coupled device* (CCD), which is the heart of modern electro-optic cameras (such as "camcorders"). CCD's consist of linear or area arrays of small (~10 μm) spots of silicon. When light is incident on the array, electrons are released from each silicon spot in proportion to the radiant energy falling on the spot.

The charge released by each spot is measured. Since the location of the silicon spots is accurately known, the pattern of released charge provides a map of the energy falling on the CCD array. When coupled with a standard camera lens, a CCD array (replacing the normal film) can record an image of the scene seen by the camera. CCD-generated false color images of underwater light fields can be seen in Voss (1989).

Theoretical understanding of the photoemissive effect came from Einstein in 1905 in a revolutionary paper (translated in Arons and Peppard, 1969) in which he introduced the concept of a photon along with its energy equation $q = h\nu$. This work was a major milestone in the history of physics, and it was primarily for his explanation of the photoemissive effect that Einstein received the Nobel Prize in 1921. Full understanding of the photovoltaic and photoconductive effects requires the modern quantum theory of the structure of matter. The photoconductive effect, for example, occurs when photons transfer electrons into the conduction band of the semiconductor, rather than completely ejecting the electrons from the material. A thorough discussion of the physics and engineering of all types of radiation detectors can be found in Budde (1983) and in Dereniak and Crowe (1984).

1.4 Directions and Solid Angles

We shall have frequent need to specify directions. In order to do this in euclidean three-dimensional space, let $\hat{i}_1$, $\hat{i}_2$, and $\hat{i}_3$ be three mutually perpendicular unit vectors that define a right-handed cartesian coordinate system. In this book we choose $\hat{i}_1$ to be in the direction that the wind is blowing over the ocean surface, (i.e. $\hat{i}_1$ points downwind), and $\hat{i}_3$ points *downward*, perpendicular to the mean position of the water surface; $\hat{i}_2$ is then in the direction given by the cross (or vector) product $\hat{i}_2 = \hat{i}_3 \times \hat{i}_1$. Our choice of a "wind-based" coordinate system simplifies the specification of sea-surface wave spectra, as must be done in Chapter 4 when we discuss radiative transfer across a wind-blown sea surface. The choice of $\hat{i}_3$ pointing downward is natural in oceanography, where depths are customarily measured as positive downward from an origin at mean sea level. In oceanographic literature one sometimes encounters the choice $\hat{i}_1$ pointing east, $\hat{i}_2$ pointing north, and $\hat{i}_3$ pointing down. This choice allows one to measure depth as positive downward, but gives a left-handed coordinate system, which is at odds with conventions in mathematics and physics. Woe unto the person who tries to do serious mathematics with a left-handed coordinate system.

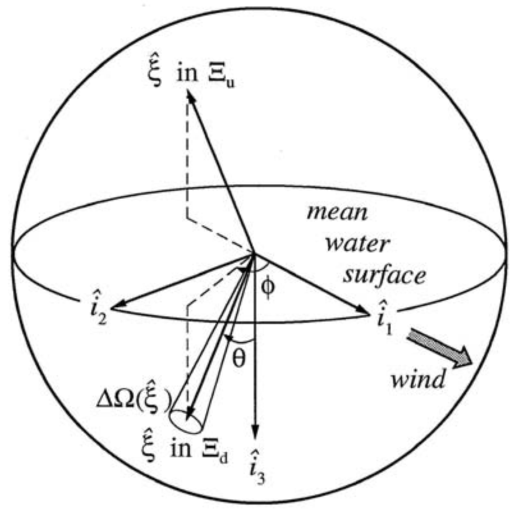

Fig. 1.3. Definition of the polar coordinates (θ,ϕ) and of the upward (Ξ_u) and downward (Ξ_d) hemispheres of directions. $\Delta\Omega(\hat{\xi})$ is an element of solid angle centered on $\hat{\xi}$.

With our choice of $\hat{i}_1$, $\hat{i}_2$, and $\hat{i}_3$, an arbitrary direction can be specified as follows. Let $\hat{\xi}$ denote a unit vector pointing in the desired direction. The vector $\hat{\xi}$ has components ξ_1, ξ_2, and ξ_3 in the $\hat{i}_1$, $\hat{i}_2$, and $\hat{i}_3$ directions, respectively. We can therefore write $\hat{\xi} = \xi_1\hat{i}_1 + \xi_2\hat{i}_2 + \xi_3\hat{i}_3$, or just $\hat{\xi} = (\xi_1, \xi_2, \xi_3)$ for notational convenience. Note that since $\hat{\xi}$ is of unit length, the components satisfy $\xi_1^2 + \xi_2^2 + \xi_3^2 = 1$.

An alternative description of $\hat{\xi}$ is given by the polar coordinates θ and ϕ, defined as shown in Fig. 1.3. The *nadir angle* θ is measured from the nadir direction $\hat{i}_3$, and the *azimuthal angle* ϕ is measured positive counterclockwise from $\hat{i}_1$, when looking toward the origin along $\hat{i}_3$ (i.e. when looking in the $-\hat{i}_3$ direction). The connection between $\hat{\xi} = (\xi_1, \xi_2, \xi_3)$ and $\hat{\xi} = (\theta, \phi)$ is obtained by inspection of Fig. 1.3:

$$\begin{aligned} \xi_1 &= \sin\theta\cos\phi \\ \xi_2 &= \sin\theta\sin\phi \\ \xi_3 &= \cos\theta\,, \end{aligned} \tag{1.7}$$

where θ and ϕ lie in the ranges $0 \le \theta \le \pi$ and $0 \le \phi < 2\pi$. The inverse transformation is

$$\begin{aligned} \theta &= \cos^{-1}(\xi_3) \\ \phi &= \tan^{-1}\left(\frac{\xi_2}{\xi_1}\right). \end{aligned} \tag{1.8}$$

The polar coordinate form of $\hat{\xi}$ could be written as $\hat{\xi} = (r,\theta,\phi)$, but since the length of $\hat{\xi}$ is always $r = 1$, we drop the radial coordinate for brevity.

Another useful description of $\hat{\xi}$ is obtained using the cosine parameter

$$\mu \equiv \cos\theta = \xi_3 . \tag{1.9}$$

The components of $\hat{\xi} = (\xi_1, \xi_2, \xi_3)$ and $\hat{\xi} = (\mu, \phi)$ are related by

$$\begin{aligned} \xi_1 &= (1-\mu^2)^{1/2}\cos\phi \\ \xi_2 &= (1-\mu^2)^{1/2}\sin\phi \\ \xi_3 &= \mu , \end{aligned} \tag{1.10}$$

with μ and ϕ in the ranges $-1 \le \mu \le 1$ and $0 \le \phi < 2\pi$. Hence a direction $\hat{\xi}$ can be represented in three equivalent ways: as (ξ_1, ξ_2, ξ_3) in cartesian coordinates, and as (θ, ϕ) or (μ, ϕ) in polar coordinates.

The dot (or scalar) product between two direction vectors $\hat{\xi}'$ and $\hat{\xi}$ can be written as

$$\hat{\xi}'\cdot\hat{\xi} \equiv |\hat{\xi}'||\hat{\xi}|\cos\psi = \cos\psi ,$$

where ψ is the angle between directions $\hat{\xi}'$ and $\hat{\xi}$, and $|\hat{\xi}|$ denotes the (unit) length of vector $\hat{\xi}$. The dot product expressed in cartesian-component form is

$$\hat{\xi}'\cdot\hat{\xi} = \xi_1'\xi_1 + \xi_2'\xi_2 + \xi_3'\xi_3 .$$

Equating these representations of $\hat{\xi}'\cdot\hat{\xi}$ and recalling Eqs. (1.7) and (1.10) leads to

$$\begin{aligned} \cos\psi &= \xi_1'\xi_1 + \xi_2'\xi_2 + \xi_3'\xi_3 \\ &= \cos\theta'\cos\theta + \sin\theta'\sin\theta\cos(\phi'-\phi) \\ &= \mu'\mu + \sqrt{1-\mu'^2}\sqrt{1-\mu^2}\cos(\phi'-\phi) . \end{aligned} \tag{1.11}$$

Equation (1.11) gives very useful connections between the various coordinate representations of $\hat{\xi}'$ and $\hat{\xi}$, and the included angle ψ.

The set of all directions $\hat{\xi}$ is called the *unit sphere* Ξ. Referring to polar coordinates, Ξ therefore represents all (θ,ϕ) values such that $0 \le \theta \le \pi$ and $0 \le \phi < 2\pi$. Two subsets of Ξ frequently employed in hydrologic optics are the *upward* (subscript *u*) and *downward* (subscript *d*) *hemispheres* of directions, Ξ_u and Ξ_d, defined by

$$\Xi_u \equiv \text{all } (\theta,\phi) \text{ such that } \pi/2 < \theta \le \pi \text{ and } 0 \le \phi < 2\pi,$$

$$\Xi_d \equiv \text{all } (\theta,\phi) \text{ such that } 0 \le \theta \le \pi/2 \text{ and } 0 \le \phi < 2\pi.$$

Solid angle

Closely allied with the specification of directions in three-dimensional space is the concept of *solid angle*, which is an extension of two-dimensional angle measurement. As illustrated in Fig. 1.4(a), the angle θ between two radii of a circle of radius *r* is

$$\theta \equiv \frac{\text{arc length}}{\text{radius}} = \frac{l}{r} \qquad \text{(rad)} .$$

The angular measure of a full circle is therefore 2π rad. In Fig. 1.4(b), a patch of area *A* is shown on the surface of a sphere of radius *r*. The boundary of *A* is traced out by a set of directions $\hat{\xi}$. The solid angle Ω of the set of directions defining the patch *A* is by definition

$$\Omega \equiv \frac{\text{area}}{\text{radius squared}} = \frac{A}{r^2} \qquad \text{(sr)} .$$

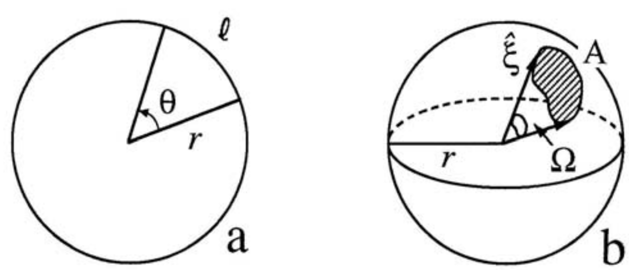

Fig. 1.4. Geometry associated with the definition of plane angle (panel *a*) and solid angle (panel *b*).

Since the area of a sphere is $4\pi r^2$, the solid angle measure of the set of all directions is $\Omega(\Xi) = 4\pi$ sr. Note that both plane angle and solid angle are independent of the radii of the respective circle and sphere.

A differential element of solid angle about the direction $\hat{\xi} = (\theta, \phi)$ is given in polar coordinate form by

$$\begin{aligned} d\Omega(\hat{\xi}) &= \sin\theta \, d\theta \, d\phi \\ &= d\mu \, d\phi \qquad \text{(sr)} . \end{aligned} \tag{1.12}$$

The last equation is correct even though $d\mu = d\cos\theta = -\sin\theta \, d\theta$. When the differential element is used in an integral and variables are changed from (θ,ϕ) to (μ,ϕ), the Jacobian of the transformation involves an absolute value. Thus an arbitrary set of directions D has a solid angle $\Omega(D)$ given symbolically by

$$\begin{aligned} \Omega(D) &= \int_{\hat{\xi} \in D} d\Omega(\hat{\xi}) \\ &= \int_{(\theta,\phi) \in D} \sin\theta \, d\theta \, d\phi \\ &= \int_{(\mu,\phi) \in D} d\mu \, d\phi \, . \end{aligned}$$

An important specific instance of D is a spherical cap of half-angle θ, i.e. all (θ',ϕ') such that $0 \le \theta' \le \theta$ and $0 \le \phi' < 2\pi$. Then

$$\Omega(D) = \int_{\phi'=0}^{2\pi} \int_{\theta'=0}^{\theta} \sin\theta' \, d\theta' \, d\phi' = 2\pi(1 - \cos\theta) \, , \tag{1.13}$$

or

$$\Omega(D) = \int_{\phi'=0}^{2\pi} \int_{\mu'=\mu}^{1} d\mu' \, d\phi' = 2\pi(1 - \mu) \, . \tag{1.14}$$

Note that Ξ_d and Ξ_u are special cases of a spherical cap (one with $\theta = \pi/2$), and that $\Omega(\Xi_d) = \Omega(\Xi_u) = 2\pi$ sr.

Delta functions

It is sometimes convenient to specify directions using the *Dirac delta function*, $\delta(\hat{\xi} - \hat{\xi}_o)$. This peculiar mathematical construction is defined (for our purposes) by

$$\delta(\hat{\xi} - \hat{\xi}_o) \equiv 0 \qquad \text{if } \hat{\xi} \neq \hat{\xi}_o\,, \tag{1.15a}$$

and

$$\int_{\Xi} f(\hat{\xi})\,\delta(\hat{\xi} - \hat{\xi}_o)\,d\Omega(\hat{\xi}) \equiv f(\hat{\xi}_o)\,. \tag{1.15b}$$

Here $f(\hat{\xi})$ is any function of direction. Note that $\delta(\hat{\xi} - \hat{\xi}_o)$ simply "picks out" the particular direction $\hat{\xi}_o$ from all directions in Ξ. Note also in Eq. (1.15b) that because the element of solid angle $d\Omega(\hat{\xi})$ has units of steradians, it follows that $\delta(\hat{\xi} - \hat{\xi}_o)$ has units of inverse steradians.

Equation (1.15) is a symbolic definition of δ. The *representation* of $\delta(\hat{\xi} - \hat{\xi}_o)$ in spherical coordinates (θ,ϕ) is

$$\delta(\hat{\xi} - \hat{\xi}_o) = \frac{\delta(\theta - \theta_o)\,\delta(\phi - \phi_o)}{\sin\theta} \qquad (\mathrm{sr}^{-1})\,, \tag{1.16}$$

where $\hat{\xi} = (\theta,\phi)$, $\hat{\xi}_o = (\theta_o,\phi_o)$, and

$$\int_0^{\pi} f(\theta)\,\delta(\theta - \theta_o)\,d\theta \equiv f(\theta_o)\,,$$

$$\int_0^{2\pi} f(\phi)\,\delta(\phi - \phi_o)\,d\phi \equiv f(\phi_o)\,.$$

Likewise, we can write

$$\delta(\hat{\xi} - \hat{\xi}_o) = \delta(\mu - \mu_o)\,\delta(\phi - \phi_o) \qquad (\mathrm{sr}^{-1})\,, \tag{1.17}$$

where

$$\int_{-1}^{1} f(\mu)\,\delta(\mu - \mu_o)\,d\mu \equiv f(\mu_o)\,.$$

Physicists just love Dirac delta functions, so a good place to learn more about them is in textbooks on modern physics; see Liboff (1980), for example.

The *Kronecker delta function*, δ_i, is a similar function for use with integer variables. We define δ_i by

$$\delta_i \equiv \begin{cases} 1 \text{ if } i = 0 \\ 0 \text{ if } i \neq 0\,, \end{cases} \tag{1.18}$$

where i is any integer. We also can write

$$\delta_{i,j} \equiv \delta_{i-j} \equiv \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j, \end{cases} \tag{1.19}$$

where i and j are any integers. We shall use whichever notation is convenient at the moment. There are no units associated with δ_i.

1.5 Geometrical Radiometry

By housing one or more radiant energy detectors in watertight assemblies and by appropriately channeling the direction of the photons arriving at the detector, we can measure the flow of radiant energy as a function of direction at any location within a water body. By adding appropriate filters to the instrument, we can also measure the wavelength dependence and state of polarization of the light field. From such measurements we can develop precise descriptions of radiative transfer in natural waters. Thus we are led to the science of *geometrical radiometry*, the union of euclidean geometry and radiometry.

Radiance

Consider an instrument designed as schematically shown in Fig. 1.5. A hole at one end of the housing or collecting tube and a system of internal light baffles allows photons that enter the hole at angles of α or less (measured from the axis of the tube) to fall onto a translucent diffusing surface of area ΔA. The diffuser makes the light field homogeneous in the region near the detector, so that it is necessary to sample only a part of the internal light field in order to measure the total energy entering the instrument. Photons of all wavelengths passing through the diffuser are then filtered so that only photons in a wavelength interval $\Delta\lambda$ centered on wavelength λ can reach the radiant energy detector. To the accuracy with which $\cos\alpha = 1$, the solid angle $\Delta\Omega$ of the hole as "seen" by any point on the diffuser surface is the same. This $\Delta\Omega$ is the solid angle subtended by the instrument. The instrument is pointing in the $-\hat{\xi}$ direction, so as to collect photons travelling in a set of directions of solid angle $\Delta\Omega$ centered on direction $\hat{\xi}$. We assume that the instrument is small compared to the scale of spatial (positional) changes in the light field, so that we can think of the instrument as being located at position $\vec{x} = (x_1, x_2, x_3) = x_1\hat{i}_1 + x_2\hat{i}_2 + x_3\hat{i}_3$ within a water body. Suitable calibration of the current (or other) output of the detector gives the amount of radiant energy ΔQ entering the

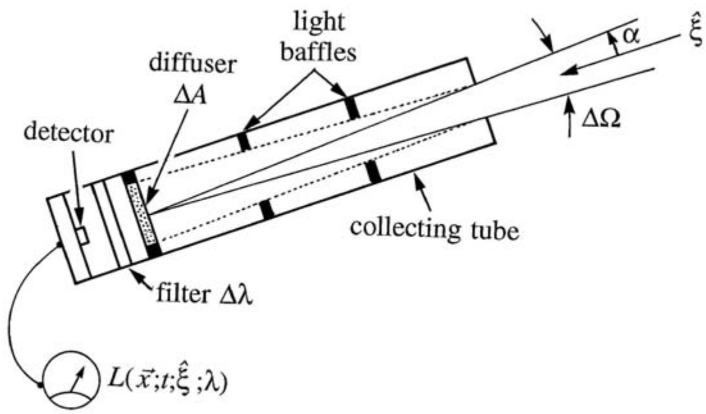

Fig. 1.5. Schematic design of an instrument for measuring unpolarized spectral radiance.

instrument during a time interval Δt centered on time t. An *operational definition* of the *unpolarized spectral radiance* is then

$$L(\vec{x};t;\hat{\xi};\lambda) \equiv \frac{\Delta Q}{\Delta t\ \Delta A\ \Delta\Omega\ \Delta\lambda} \qquad (\mathrm{J\ s^{-1}\ m^{-2}\ sr^{-1}\ nm^{-1}})\,. \tag{1.20}$$

In practice the intervals Δt, ΔA, $\Delta\Omega$, and $\Delta\lambda$ are taken small enough to get a useful resolution of L over the various parameter domains, but not so small as to encounter diffraction effects or fluctuations from photon shot noise at low light levels. Typical values are $\Delta t \sim 10^{-2}$ to 10^3 s (depending on whether one wishes an "instantaneous" measurement or wishes to average out sea surface wave effects), $\Delta A \sim 10^{-4}$ to 10^{-3} m^2, $\Delta\Omega \sim 0.01$ to 0.1 sr, and $\Delta\lambda \sim 1$ to 10 nm. We shall always let $\hat{\xi}$ *denote the direction of photon travel*. The *viewing direction*, the direction an instrument is pointed to detect $L(\vec{x};t;\hat{\xi};\lambda)$ is then $-\hat{\xi} = (\pi-\theta,\ \phi+2\pi)$.

In the conceptual limit of infinitesimal parameter intervals, the spectral radiance is given by

$$L(\vec{x};t;\hat{\xi};\lambda) = \frac{\partial^4 Q}{\partial t\,\partial A\,\partial\Omega\,\partial\lambda} \qquad (\mathrm{W\ m^{-2}\ sr^{-1}\ nm^{-1}})\,. \tag{1.21}$$

Spectral radiance is the fundamental radiometric quantity of interest in hydrologic optics. It specifies the spatial ($\vec{x}$), temporal (t), directional ($\hat{\xi}$), and wavelength (λ) structure of the light field. As we shall see, *all other radiometric quantities can be derived from L*. However, both because of instrumental difficulties and because such complete information often is not needed for specific applications, the most commonly measured radiometric quantities are various irradiances.

Irradiance

If the collecting tube is removed from the instrument of Fig. 1.5, then photons from an entire hemisphere of directions can reach the detector, as illustrated in Fig. 1.6. Such an instrument, when pointed "straight up" (in the $-\hat{i}_3$ direction) so as to detect photons headed *downward* (all $\hat{\xi}$ in Ξ_d) measures the *spectral downward plane irradiance* E_d:

$$E_d(\vec{x};t;\lambda) \equiv \frac{\Delta Q}{\Delta t\,\Delta A\,\Delta\lambda} \qquad (\mathrm{W\ m^{-2}\ nm^{-1}})\,. \tag{1.22}$$

Implicit in this definition is the assumption that each point of the collector surface is equally sensitive to photons incident onto the surface from any angle. If this is the case, however, the collector *as a whole* is *not* equally sensitive to all photons headed in downward directions. Imagine a collimated beam of light headed straight downward (e.g. from the sun straight overhead). This beam, assumed to be larger than the collector surface, sees the full area ΔA of the collector surface. However, the same

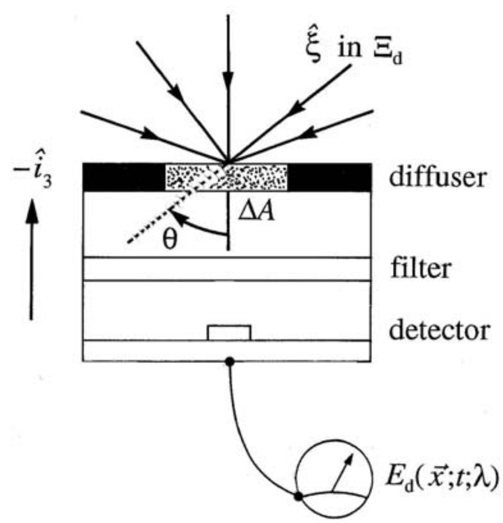

Fig. 1.6. Schematic design of an instrument for measuring spectral plane irradiance.

large beam traveling at an angle θ relative to the instrument axis sees a collector surface of effective area $\Delta A|\cos\theta|$ (the area ΔA as projected onto a plane perpendicular to the beam direction). Otherwise identical light beams therefore generate detector responses that are proportional to the cosines of the incident photon directions. Such instruments are called *cosine collectors.*

The *cosine law for irradiance* is simply the statement that a collimated beam of photons intercepting a plane surface produces an irradiance that is proportional to the cosine of the angle between the photon directions and the normal to the collector surface.

Since the instrument of Fig. 1.6 collects photons traveling in all downward directions, but with each photon's contribution weighted by the cosine of the photon's incident angle θ, the instrument is in essence integrating $L(\vec{x};t;\hat{\xi};\lambda)|\cos\theta|$ over all downward directions. The spectral downward plane irradiance is therefore related to the spectral radiance by

$$
\begin{aligned}
E_{\mathrm{d}}(\vec{x};t;\lambda) &= \int_{\hat{\xi}\,\in\,\Xi_{\mathrm{d}}} L(\vec{x};t;\hat{\xi};\lambda)\,|\cos\theta|\; d\Omega(\hat{\xi}) \\
&= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} L(\vec{x};t;\theta,\phi;\lambda)|\cos\theta|\sin\theta\; d\theta\; d\phi\,.
\end{aligned}
\tag{1.23}
$$

If the same instrument is oriented *downward*, so as to detect photons heading *upward*, then the quantity being measured is the *spectral upward plane irradiance* E_{u}:

$$
\begin{aligned}
E_{\mathrm{u}}(\vec{x};t;\lambda) &= \int_{\hat{\xi}\,\in\,\Xi_{\mathrm{u}}} L(\vec{x};t;\hat{\xi};\lambda)\,|\cos\theta|\; d\Omega(\hat{\xi}) \\
&= \int_{\phi=0}^{2\pi}\int_{\theta=\pi/2}^{\pi} L(\vec{x};t;\theta,\phi;\lambda)|\cos\theta|\sin\theta\; d\theta\; d\phi\,.
\end{aligned}
\tag{1.24}
$$

Note that it is necessary to take the absolute value of $\cos\theta$ in Eq. (1.24) since, with our choice of coordinates, $\cos\theta < 0$ for $\hat{\xi}$ in Ξ_{u}. The absolute value operation was superfluous in Eq. (1.23).

Now consider an instrument that is designed to be *equally* sensitive to all photons headed in the downward direction. Such an instrument is shown in Fig. 1.7. The spherical shape of the diffuser insures that the instrument is equally sensitive to photons from any direction. If each point on the diffuser surface behaves like a cosine collector, then the effective area of the collector is $\Delta A = \pi r^2$, where r is the radius of the diffuser. The

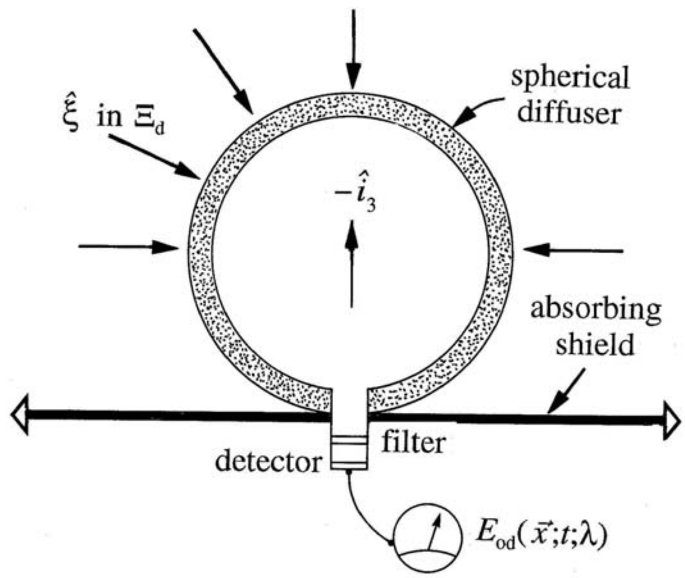

Fig. 1.7. Schematic design of an instrument for measuring spectral scalar irradiance.

large shield blocks upward-traveling photons. The upper surface of the shield is assumed to be completely absorbing, so that it cannot reflect downward traveling photons back upward into the diffuser. This instrument, when oriented upward as shown in Fig. 1.7, measures the *spectral downward scalar irradiance* E_{od}, which is related to the spectral radiance by

$$\begin{aligned} E_{od}(\vec{x};t;\lambda) &= \int_{\hat{\xi}\,\in\,\Xi_d} L(\vec{x};t;\hat{\xi};\lambda)\, d\Omega(\hat{\xi}) \\ &= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} L(\vec{x};t;\theta,\phi;\lambda)\,\sin\theta\, d\theta\, d\phi\,. \end{aligned} \tag{1.25}$$

If the instrument of Fig. 1.7 is inverted, so as to collect only upward traveling photons, then it measures the *spectral upward scalar irradiance* E_{ou}. If this shield is removed, photons traveling in all directions are collected. The quantity then measured is the *spectral total scalar irradiance* E_o:

$$\begin{aligned} E_o(\vec{x};t;\lambda) &\equiv \int_{\hat{\xi}\,\in\,\Xi} L(\vec{x};t;\hat{\xi};\lambda)\, d\Omega(\hat{\xi}) \\ &= E_{od}(\vec{x};t;\lambda) + E_{ou}(\vec{x};t;\lambda)\,. \end{aligned} \tag{1.26}$$

Other possible instrument designs are discussed in Højerslev (1975). However, the three shown in Figs. 1.5 to 1.7 are sufficient for most of the needs of hydrologic optics. Such instruments are commercially available.

The *spectral vector irradiance* $\vec{E}$ is defined as

$$\vec{E}(\vec{x};t;\lambda) \equiv \int_{\hat{\xi} \in \Xi} L(\vec{x};t;\hat{\xi};\lambda)\, \hat{\xi}\, d\Omega(\hat{\xi}) \,. \tag{1.27}$$

The direction $\hat{\xi}$ can be written in component form as

$$\hat{\xi} = (\hat{\xi}\cdot\hat{i}_1)\hat{i}_1 + (\hat{\xi}\cdot\hat{i}_2)\hat{i}_2 + (\hat{\xi}\cdot\hat{i}_3)\hat{i}_3 \,,$$

where, for example, $\hat{\xi}\cdot\hat{i}_1$ is the dot product of vectors $\hat{\xi}$ and $\hat{i}_1$. Thus the vector irradiance can be written as

$$\begin{aligned}
\vec{E} &= \left[\int_{\Xi} L(\vec{x};t;\hat{\xi};\lambda)\,(\hat{\xi}\cdot\hat{i}_1)\, d\Omega(\hat{\xi})\right]\hat{i}_1 \\
&+ \left[\int_{\Xi} L(\vec{x};t;\hat{\xi};\lambda)\,(\hat{\xi}\cdot\hat{i}_2)\, d\Omega(\hat{\xi})\right]\hat{i}_2 \\
&+ \left[\int_{\Xi} L(\vec{x};t;\hat{\xi};\lambda)\,(\hat{\xi}\cdot\hat{i}_3)\, d\Omega(\hat{\xi})\right]\hat{i}_3 \\
&\equiv (\vec{E})_1\,\hat{i}_1 + (\vec{E})_2\,\hat{i}_2 + (\vec{E})_3\,\hat{i}_3 \,.
\end{aligned}$$

The *vertical component* of the vector irradiance can be expressed in terms of the plane irradiances:

$$\begin{aligned}
(\vec{E})_3 &= \hat{i}_3\cdot\vec{E} \\
&= \int_{\Xi} L(\vec{x};t;\hat{\xi};\lambda)\cos\theta\, d\Omega(\hat{\xi}) \\
&= E_d - E_u \,.
\end{aligned} \tag{1.28}$$

In developing Eq. (1.28), we have recalled that $\hat{\xi}\cdot\hat{i}_3 = \cos\theta$, and we have noted that $\cos\theta > 0$ in Ξ_d and $\cos\theta < 0$ in Ξ_u. The quantity $E_d - E_u$ is commonly called the *net* (downward) *irradiance*. This net irradiance often is called the "vector" irradiance, although strictly speaking it is only the vertical component of the vector irradiance.

Examples

Consider an *isotropic*, or directionally uniform, radiance distribution: $L(\vec{x};t;\hat{\xi};\lambda) \equiv L_o(\vec{x};t;\lambda)$ for all $\hat{\xi}$ in Ξ. Then by Eq. (1.23), the downward plane irradiance is

$$E_d(\vec{x};t;\lambda) = \int_0^{2\pi}\int_0^{\pi/2} L_o(\vec{x};t;\lambda)\cos\theta\sin\theta\, d\theta\, d\phi$$
$$= \pi L_o(\vec{x};t;\lambda)\,.$$

In the last equation, π carries units of steradian from the integration over solid angle; thus E_d has units of irradiance when L_o has units of radiance. Likewise, $E_u = \pi L_o$, so that the net irradiance is $E_d - E_u = 0$. The scalar irradiance E_{od} is given by Eq. (1.25):

$$E_{od}(\vec{x};t;\lambda) = \int_0^{2\pi}\int_0^{\pi/2} L_o(\vec{x};t;\lambda)\sin\theta\, d\theta\, d\phi$$
$$= 2\pi L_o(\vec{x};t;\lambda)\,.$$

In the same manner we find $E_{ou} = 2\pi L_o$, so that the total scalar irradiance $E_o = 4\pi L_o$.

Radiance distributions at great depth are *approximately* described by an elliptical distribution (see Sections 5.9 and 9.6):

$$L(\vec{x};t;\hat{\xi};\lambda) = \frac{L_o(\vec{x};t;\lambda)}{1 - \epsilon\cos\theta}\,,$$

where ϵ, the eccentricity of the ellipse, is a constant satisfying $0 < \epsilon < 1$. For light fields in natural waters, $\epsilon > 0.9$. The corresponding E_d is

$$E_d = 2\pi L_o \int_0^{\pi/2} \frac{1}{1 - \epsilon\cos\theta}\cos\theta\sin\theta\, d\theta\,,$$

which after a difficult integration becomes

$$E_d = -\frac{2\pi L_o}{\epsilon^2}\left[\epsilon + \ln(1 - \epsilon)\right].$$

Note that $E_d > 0$ for all allowed values of ϵ. In a similar fashion we find

$$E_u = \frac{2\pi L_o}{\epsilon^2}\left[\epsilon - \ln(1 + \epsilon)\right],$$

and so the net downward irradiance is

$$E_d - E_u = \frac{2\pi L_o}{\epsilon^2}\left[-2\epsilon + \ln\left(\frac{1 + \epsilon}{1 - \epsilon}\right)\right].$$

The scalar irradiances are

$$E_{od} = 2\pi L_o \int_0^{\pi/2} \frac{1}{1 - \epsilon \cos\theta} \sin\theta \, d\theta$$

$$= -\frac{2\pi L_o}{\epsilon} \ln(1 - \epsilon) ,$$

$$E_{ou} = \frac{2\pi L_o}{\epsilon} \ln(1 + \epsilon) ,$$

and

$$E_o = \frac{2\pi L_o}{\epsilon} \ln\left(\frac{1 + \epsilon}{1 - \epsilon} \right) .$$

Note (after application of l'Hospital's rule) that the irradiances associated with the elliptical radiance distribution approach those of the isotropic distribution as ϵ approaches 0.

Intensity

Another family of radiometric quantities can be defined from the measurements employed in the operational definitions (1.20) and (1.22). The *spectral intensity I* is defined as

$$I(\vec{x};t;\hat{\xi};\lambda) = \frac{\Delta Q}{\Delta t \, \Delta\Omega \, \Delta\lambda} \qquad (\mathrm{W\ sr^{-1}\ nm^{-1}}) , \tag{1.29}$$

or

$$I(\vec{x};t;\hat{\xi};\lambda) = \int_{\Delta A} L(\vec{x};t;\hat{\xi};\lambda) \, dA . \tag{1.30}$$

In Eq. (1.30), ΔA is the surface of the collector which sees the solid angle $\Delta\Omega$, and dA is an element of area. Just as for irradiance, we can define various plane, scalar and vector intensities by insertion of the appropriate factors into the integrand of Eq. (1.30), which as written represents the scalar intensity I_o. The concept of intensity is useful in the radiometry of point light sources (see *H.O.* Section 2.9), but it has not found much application in hydrologic optics except in the definition of the volume scattering function, as will be seen in Eqs. (3.4) and (5.3).

Terminology and notation

The adjective "spectral," as in "spectral radiance," in colloquial use can mean either "as a function of wavelength" or "per unit wavelength interval." Committees on international standards recommend an argument λ, e.g. $L(\lambda)$, for the first meaning and a subscript λ, e.g. L_λ, for the second meaning (Meyer-Arendt, 1968). However, this subscript convention is seldom used in hydrologic optics, perhaps because the symbols already are cluttered with subscripts. If the possibility of confusion exists, one can use "monochromatic" to signify the second meaning; thus "monochromatic radiance $L(\vec{x};t;\hat{\xi};\lambda)$" denotes the "radiance per unit wavelength interval (e.g. per nanometer) at the particular wavelength λ." The adjective spectral and argument λ are often omitted for brevity, although strictly speaking, a term without the adjective "spectral" refers to a quantity integrated or measured over a broad band of wavelengths, as for example the radiance over the visible band:

$$L(\vec{x};t;\hat{\xi}) = \int_{400\text{ nm}}^{700\text{ nm}} L(\vec{x};t;\hat{\xi};\lambda)\, d\lambda \qquad (\text{W m}^{-2}\text{ sr}^{-1})\,.$$

Most radiative transfer *theory* assumes the energy to be monochromatic, but most *measurements* are made over a fairly wide wavelength band, which complicates the comparison of theory and observation.

Since sampling times Δt are generally long compared to the time ($\sim 10^{-6}$ s) required for the light field to reach steady state after a change in the environment, time-independent radiative transfer theory is usually sufficient for hydrologic optics studies. In this case, the time is implicitly understood and the argument t can be omitted. Finally, horizontal variations in the environment and in the optical properties of natural water bodies (on a scale of tens to thousands of meters) are usually much less than vertical variations, so that to a good approximation underwater that light fields depend spatially only on the depth x_3. Thus, for example, we often can refer to just "the radiance $L(x_3;\theta,\phi)$" without generating confusion.

Although this book uses the terminology and notation commonly seen in the current hydrologic optics literature, much published work uses a different nomenclature. Prior to the late 1970's, different symbols were often employed for the radiometric concepts just defined. This historic notation is seen, for example, in *Hydrologic Optics* (Preisendorfer, 1976). Table 1.5 compares the recommended and historic symbols.

Much of the work in atmospheric, astrophysical, and biomedical optics, and in neutron transport theory, is relevant to hydrologic optics. However, to a considerable extent, these fields have developed their own nomenclature. For example, in atmospheric and astrophysical optics, radiance is often called "intensity" or "specific intensity" and given the symbol I; this use of "intensity" is not be confused with intensity as defined above. The classic text by van de Hulst (1957) uses "intensity" for irradiance. The letter "I" often is used as the symbol for irradiance. The word "flux" is sometimes used to mean irradiance and is sometimes used to

Table 1.5. Terms, units, and symbols for radiometric quantities commonly used in hydrologic optics. The quantities as shown represent broadband measurements. For narrow band (monochromatic) measurements, add the adjective "spectral" to the term, add nm^{-1} to the units, and add a wavelength index λ to the symbol, e.g. spectral radiance, L_λ or $L(\lambda)$, with units of W m^{-2} sr^{-1} nm^{-1}. PAR is always broadband.

Quantity	SI Units	Recommended symbol	Historic symbol
radiant energy	J	Q	U
radiant power	W	Φ	P
radiant intensity	W sr^{-1}	I	J
radiance	W m^{-2} sr^{-1}	L	N
plane irradiance	W m^{-2}	E	H
downward plane irradiance	W m^{-2}	E_d	$H(-)$
upward plane irradiance	W m−2	E_u	$H(+)$
scalar irradiance	W m^{-2}	$\mathring{E}$ or E_o	h
downward scalar irradiance	W m^{-2}	$\mathring{E}_d$ or E_{od}	$h(-)$
upward scalar irradiance	W m^{-2}	$\mathring{E}_u$ or E_{ou}	$h(+)$
vector irradiance	W m^{-2}	$\vec{E}$	$\vec{H}$
(vertical) net irradiance	W m^{-2}	$E_d - E_u$	—
radiant exitance	W m^{-2}	M	W
photosynthetically available radiation	photons s^{-1} m^{-2}	PAR or E_{PAR}	—

mean power. Those who use "flux" for power generally use "flux density" for irradiance. We shall avoid the ambiguity associated with "flux" simply by avoiding the term. Matters are further complicated by the occasional misuse of photometric terms such as "brightness" and "luminance" for radiometric quantities; these matters are discussed in Chapter 2.

It is occasionally convenient to distinguish conceptually between photons leaving a surface and photons arriving at a surface. *Field radiance* L^- refers to the radiance of photons arriving at a surface; this is the quantity *measured* by a radiance meter like that of Fig. 1.5. *Surface radiance* L^+ is the radiance *attributed* to a real or imaginary surface emitting photons. *Irradiance* E refers to photons incident *onto* a surface; photons *leaving* a surface are denoted by *radiant exitance* or *emittance* M. Likewise, intensity can be subdivided into field intensity I^- and surface intensity I^+. Figure 1.8 summarizes this hierarchy of radiometric concepts.

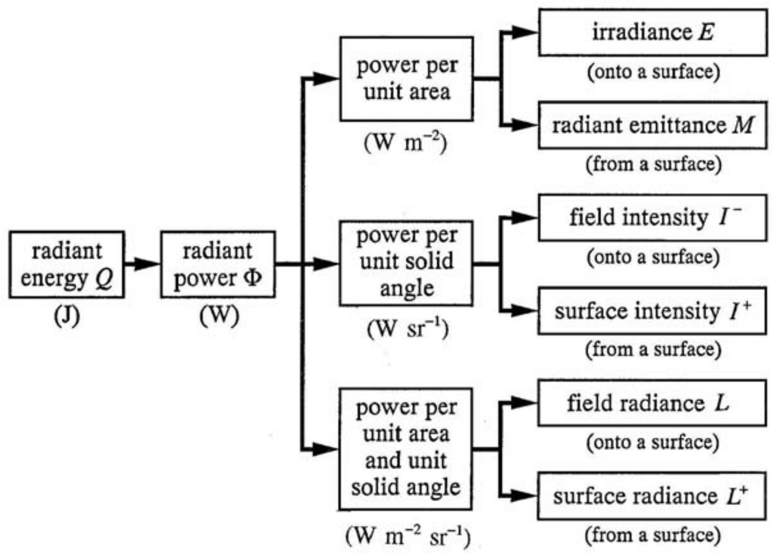

Fig. 1.8. Hierarchy of radiometric concepts.

Spectral quantities are sometimes expressed in terms of a unit frequency interval rather than in terms of a unit wavelength interval. To establish the conversion, consider the radiant energy $Q(\lambda)d\lambda$ contained in a wavelength interval $d\lambda$. The same amount of energy is contained in a corresponding frequency interval $Q(\nu)d\nu$. Since an increase in wavelength ($d\lambda > 0$) implies a decrease in frequency ($d\nu < 0$), and vice versa, we can write

$$Q(\lambda)\, d\lambda \;=\; -Q(\nu)\, d\nu\,.$$

Using $\lambda = c/\nu$ we then get

$$Q(\nu) \;=\; -Q(\lambda)\frac{d\lambda}{d\nu} \;=\; \frac{c}{\nu^2}Q(\lambda) \;=\; \frac{\lambda^2}{c}Q(\lambda)\,,$$

which is the desired connection between $Q(\lambda)$ and $Q(\nu)$, or between any other radiometric quantities. A wavelength interval $d\lambda$ corresponds to a frequency interval $d\nu$ of size

$$d\nu \;=\; \frac{c}{\lambda^2}\, d\lambda\,. \tag{1.31}$$

The *wavenumber* $\kappa \equiv 1/\lambda$ is also sometimes used (the value usually given in inverse centimeters, cm^{-1}). An analysis parallel to that just given yields

$$d\kappa = \frac{1}{\lambda^2}\, d\lambda\,. \tag{1.32}$$

Radiance invariance law

Radiance is distinguished by the property that it does not change along a photon path in a vacuum. This fact may be understood by means of Fig. 1.9. Panel *a* of the figure depicts a radiance meter RM directed at a surface S at the end of a clear path of sight of length r. The surface S is normal to the line of sight and has a uniform surface radiance L_o over its extent in the direction of RM. The meter RM has its field of view completely filled by S. The resultant field radiance reading is L_r. We now show that under these conditions, $L_o = L_r$ for every r.

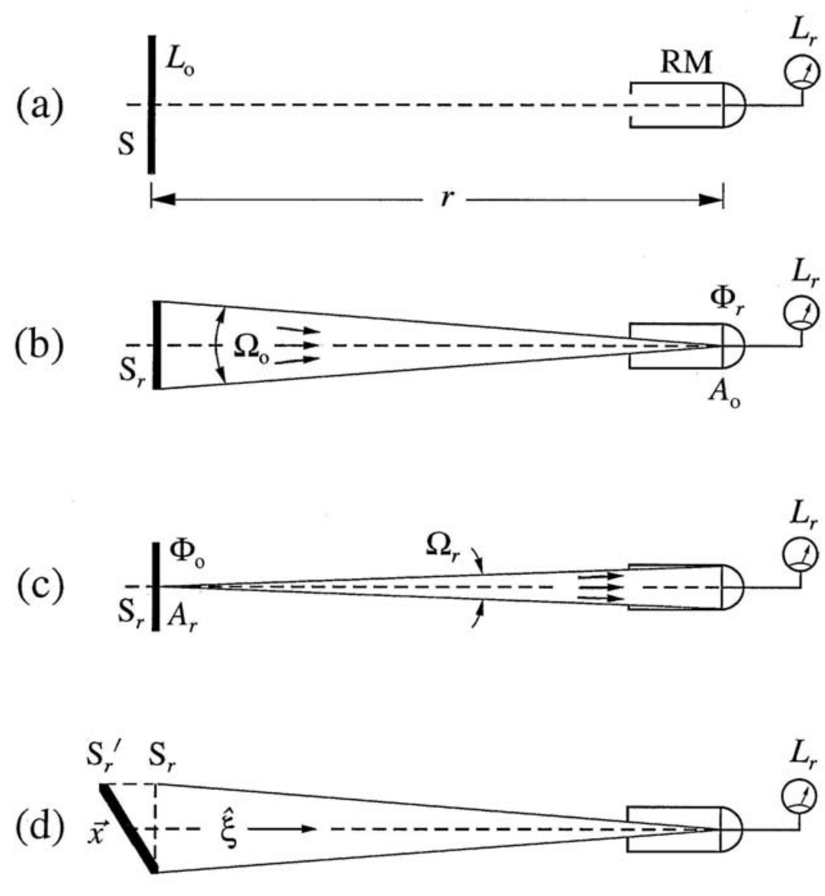

Fig. 1.9. Geometry used in proving the radiance invariance law.

The basic idea of the proof is to examine the same diagram from two distinct points of view, which are shown in panels *b* and *c* of Fig. 1.9. Consider panel *b* first. Here the radiance meter's reading L_r is seen to be the field radiance quotient $\Phi_r/A_o\Omega_o$, where Φ_r is the radiant power originating from S and incident on the collecting surface of area A_o, and which is funneled through the solid angle of magnitude Ω_o, defined as shown. On the other hand, panel *c* views this same situation as a power Φ_o emitted from those points of S_r within RM's field of view, as channeled through a solid angle Ω_r into the collector surface A_o. The emitting surface S_r comprising RM's field of view is of variable area A_r (fixed for each *r*), and the emitted power from each point of S_r is within the bundle of directions of solid angle Ω_r defined as shown. Hence L_o is the quotient $\Phi_o/A_r\Omega_r$. Since the same radiant power is received by the detector regardless of whether we take the viewpoint of panel *b* or panel *c*, it follows that $\Phi_o = \Phi_r$. Further, from the definition of solid angle, we have the geometric observation that

$$\frac{A_o A_r}{r^2} = \Omega_r A_r = A_o \Omega_o .$$

In optical engineering, the quantity $A_o\,\Omega_o = A_r\Omega_r$ is called the *throughput*. On the basis of these two facts, we see that by virtue of the defining equations

$$L_r = \frac{\Phi_r}{A_o \Omega_o}$$

and

$$L_o = \frac{\Phi_o}{A_r \Omega_r} ,$$

it follows that

$$L_o = L_r \tag{1.33}$$

for all $r \geq 0$. The radiance invariance law will be generalized in Section 4.2 to the case of propagation through material media.

Had we wished to be extremely precise, we could have distinguished between field and surface radiances in the preceding proof. For example, we could write Eq. (1.33) as $L_o^+ = L_r^-$, since L_o is a surface radiance and L_r is a field radiance. Such notational rigor is seldom demanded in hydrologic optics.

Consider the pleasant situation of sitting in front of a fireplace as you read this book. You look into the fire and perceive a certain "brightness" (in essence, radiance; see Chapter 2), and you feel a certain amount of heat coming from the fire (in essence, irradiance; it is energy absorbed per unit time per unit area of your body that warms you). If you move your chair farther away from the fire, it appears smaller but the flames are just as bright; this is the radiance invariance law in action. However, you feel less heat coming from the fire; this is a consequence of the inverse square law for irradiance. This simple situation holds true as long as our radiometer – in this case, our eye – can resolve a finite solid angle for the fire. If we move so far away from the fire that our eye sees it as a true point source of light, then our eye responds to the irradiance received from the source. Thus the fire eventually fades from view as we continue to move farther away.

Assigning surface radiances

We can *assign* a radiance to a point on a distant surface by means of the radiance invariance law. In panel *d* of Fig. 1.9 let $\vec{x}$ be a point, say, on the surface of a cloud on a clear day. The narrow field of view of the radiance meter closes down arbitrarily finely on a small element of surface S_r' about $\vec{x}$ so that the power leaving S_r' within the indicated solid angle also crosses its projection S_r normal to the direction $\hat{\xi}$ of the line of sight. When this is so, then by definition the surface radiance L_o of S_r' at $\vec{x}$ along $\hat{\xi}$ is also that of S_r along $\hat{\xi}$ where the line of sight crosses S_r. The radiance invariance law then allows us to assign the distantly measured field radiance L_r to this surface radiance L_o of S_r *when there is no loss of photons incurred between* S_r *and* RM. When the path of sight between $\vec{x}$ and RM is not perfectly transparent, then the intervening scattered and absorbed light along the path must be accounted for in determining the field radiance L_r at RM. How this is done is one of the main problems addressed in the present book.

As an application of the preceding ideas, we can assign a surface radiance to the sun. The sun's disk subtends a half angle of $\theta_s = 4.64\times10^{-3}$ rad at the mean distance of the earth's orbit. By Eq. (1.13), the solid angle of the solar disk is then

$$\Omega_s = 2\pi(1 - \cos\theta_s) = 6.76\times10^{-5} \text{ sr} .$$

In Fig. 1.1 we see that the maximum value of the sun's spectral irradiance $E_s(\lambda)$ is approximately 2 W m^{-2} nm^{-1}. The corresponding spectral radiance $L_s(\lambda)$ is then

$$L_s(\lambda) = \frac{E_s(\lambda)}{\Omega_s} \approx 3\times10^4 \text{ W m}^{-2} \text{ sr}^{-1} \text{ nm}^{-1} .$$

By way of comparison, typical clear-sky spectral radiances in directions away from the sun are $L_{sky} \sim 0.1$ W m^{-2} sr^{-1} nm^{-1}. The total solar radiance over all wavelengths is

$$L_s = \frac{E_s}{\Omega_s} = 2.02\times10^7 \ \mathrm{W\ m^{-2}\ sr^{-1}}\,.$$

1.6 Photosynthetically Available Radiation

Photosynthesis is a quantum process. That is to say, it is the *number* of available photons rather than their total energy that is relevant to the chemical transformations. This is because a photon of, say, wavelength 400 nm, *if absorbed* by cholorphyll, induces the same chemical change as does a less energetic photon of wavelength 700 nm. (However, photons of different wavelengths are not equally likely to be absorbed; see Section 3.7.) Only a part of the photon's energy goes into photosynthesis; the excess appears as heat or is re-radiated. Moreover, chlorophyll is equally able to absorb a photon regardless of the photon's direction of travel.

Now recall that the spectral total scalar irradiance $E_o(\vec{x};\lambda)$ is the total radiant power per square meter at wavelength λ coursing through point $\vec{x}$ owing to photons traveling in all directions. By Eq. (1.1), the number of photons generating E_o is $E_o\lambda/hc$. Chlorophyll can absorb and utilize photons of near ultraviolet and visible wavelengths. Therefore, in studies of phytoplankton biology, the relevant measure of the underwater light field is the *photosynthetically available radiation*, *PAR* or E_{PAR}, defined by

$$PAR(\vec{x}) \equiv \int_{350\,\mathrm{nm}}^{700\,\mathrm{nm}} \frac{\lambda}{hc} E_o(\vec{x};\lambda)\, d\lambda \qquad (\mathrm{photons\ s^{-1}\ m^{-2}})\,. \tag{1.34}$$

Note that *PAR* is by definition a broadband quantity; there is no "spectral *PAR*." Bio-optical literature often states *PAR* values in units of mol photons $\mathrm{s^{-1}\ m^{-2}}$ or einst $\mathrm{s^{-1}\ m^{-2}}$; one einstein is one mole of photons (6.023×10^{23} photons).

Morel and Smith (1974) found that over a wide variety of water types from very clear to turbid, the conversion factor for energy to photons varies by only ±10% about the value

$$2.5\times10^{18}\ \mathrm{photons\ s^{-1}\ W^{-1}} = 4.2\ \mu\mathrm{einst\ s^{-1}\ W^{-1}}\,.$$

PAR is often estimated using only the visible wavelengths, 400-700 nm. Omission of the near-UV band is usually permissible since wavelengths less than 400 nm are rapidly absorbed near the water surface, except in very clear waters. Instruments for the direct measurement of *PAR*, called *quanta* *meters,* can be constructed along the lines of Fig. 1.7 by the incorporation of suitable wavelength filters. Such instruments are commercially available. Engineering details can be found in Jerlov and Nygård (1969) and in Kirk (1983), which also discusses *PAR* and photosynthesis in great detail.

1.7 Physical Foundations

In order to complete our discussion of radiometry, we need to comment briefly on its connection with the mainland of physics. For pedagogic purposes, we have viewed a light beam as composed of many photons. However, as mentioned in Section 1.1, we can equivalently view light as a propagating transverse wave composed of time varying electric and magnetic fields. These fields are related by Maxwell's equations, which themselves are grounded in the laws of quantum electrodynamics – the most successful physical theory ever devised.

Let $\vec{E}(\vec{x};t)$ and $\vec{B}(\vec{x};t)$ be respectively the instantaneous electric and magnetic fields associated with the light field at location $\vec{x}$. Then it is shown in textbooks on electromagnetic theory (e.g. Griffiths, 1981) that the *Poynting vector*

$$\vec{S}(\vec{x};t) \equiv \frac{1}{\mu_o} \vec{E}(\vec{x};t) \times \vec{B}(\vec{x};t)$$

is the instantaneous power transferred across a surface of unit area at location $\vec{x}$, if the surface is oriented perpendicular to the direction of $\vec{S}$. Here $\mu_o = 4\pi \times 10^{-7}$ kg m s^{-2} A^{-2} is the permeability of free space. $\vec{E}$ has units of volts per meter (kg m s^{-3} A^{-1}) and $\vec{B}$ has units of $\vec{E}/c$ (kg s^{-2} A^{-1}; c is the speed of light). Thus $\vec{S}$ has units of kg s^{-3}, i.e. W m^{-2}, the units of irradiance. The direction of $\vec{S}$ is the direction $\hat{\xi}$ of photon travel. A radiometric sensor of area ΔA making a measurement for a time Δt responds to the energy

$$\Delta Q = \int_{\Delta t} \int_{\Delta A} |\vec{S}(\vec{x};t)| \, dA \, dt \qquad \text{(J)},$$

where the area integration is over all points $\vec{x}$ of the sensor surface and $|\vec{S}|$ is the magnitude of $\vec{S}$. The connection between electromagnetic theory and the definitions of the radiometric quantities is thus established. A rigorous treatment of this connection is given in Preisendorfer (1965).

Incidentally, the energy density (energy per cubic meter) of an electromagnetic field is

$$Q_{\text{den}} = \frac{|\vec{S}|}{c} \qquad (\text{J m}^{-3} = \text{kg s}^{-2}\ \text{m}^{-1}),$$

and the momentum density is

$$P_{\text{den}} = \frac{|\vec{S}|}{c^2} \qquad (\text{kg m s}^{-1}\ \text{m}^{-3} = \text{kg s}^{-1}\ \text{m}^{-2}).$$

Note that these electromagnetic-field densities stand in the same relation to each other, namely $Q_{\text{den}} = P_{\text{den}}c$, as do the energy and momentum of an individual photon: $q = hc/\lambda = pc$.

1.8 Problems

"Be ye doers of the word and not hearers only, lest ye deceive yourselves."
— somewhere in the Bible

1.1. Compute the solid angles of the 10° by 10° quadrilateral regions bounded by

(a) $30° \le \theta \le 40°$ and $20° \le \phi \le 30°$
(b) $70° \le \theta \le 80°$ and $20° \le \phi \le 30°$
(c) $70° \le \theta \le 80°$ and $60° \le \phi \le 70°$

1.2. The earth's surface has an area of about 1.97×10^8 square miles. The United States covers about 3.68×10^6 square miles, including Alaska and Hawaii. What is the total solid angle subtended by the United States as seen from the center of the earth?

1.3. What is the value of the total (all wavelengths) emittance M at the surface of the sun? The sun's radius is 6.951×10^8 m, and the mean radius of the earth's orbit is 1.496×10^{11} m.

1.4. Roughly how many visible-wavelength photons from a single, barely visible star enter a human eye each second on a clear night?

1.5. The sky radiance distribution on a heavily overcast day is approximately described by the *cardioidal* radiance distribution:

$$L(\theta,\phi) = L_o(1 + 2\cos\theta), \qquad 0 \le \theta \le \frac{\pi}{2}. \tag{1.35}$$

(Remember, you are looking upward, but the photons are heading downward, and θ = 0 is straight down.) Compute E_d and E_{od} for the cardioidal distribution.

1.6. Moscow, Russia is at latitude 55.9°N and longitude 37.5°E; São Paulo, Brazil is at 23.5°S, 46.7°W. What are the unit vectors $\hat{\xi}_M$ and $\hat{\xi}_{SP}$ that describe the directions of Moscow and São Paulo, in a coordinate system based at the center of the earth. Let $\theta = 0$ be the north pole and $\theta = 180°$ be the south pole; $\phi = 0$ is the Greenwich meridian, and east longitude is $\phi > 0$. Show both forms $\hat{\xi} = (\xi_1, \xi_2, \xi_3)$ and (μ,ϕ). What is the included angle ψ between $\hat{\xi}_M$ and $\hat{\xi}_{SP}$?

1.7. Consider the radiance distribution

$$L(\theta,\phi) = L_o(2 + 10\cos^2\theta)\sin^2\phi \qquad \text{if } 0 \le \theta \le \pi/2$$

$$L(\theta,\phi) = L_o(1 + \sin^2\theta)\sin^2\phi \qquad \text{if } \pi/2 \le \theta \le \pi.$$

Compute E_d, E_u, E_{od}, E_{ou}, E_o and $\vec{E}$ for this distribution.

1.8. As a crude approximation, the sky radiance distribution on a clear day can be represented as a collimated direct solar beam plus an isotropic diffuse sky radiance. Let (θ_s,ϕ_s) be the direction of the direct beam, which contributes a fraction f $(0 \le f \le 1)$ to the total sun-plus-sky irradiance E_d. The diffuse sky radiance then contributes $1-f$ of the total irradiance. Such a radiance distribution can be written as

$$L(\theta,\phi) = C\left[f\,\delta(\cos\theta - \cos\theta_s)\,\delta(\phi - \phi_s) + \frac{1-f}{\pi}\right],$$

where C is a constant that sets the overall magnitude of L.
(a) What are the dimensions of C?
(b) Compute E_d and E_{od} in terms of C.
(c) Modify the above equation so that $L(\theta,\phi)$ represents a direct beam plus a cardioidal background sky. Such a radiance distribution might be a better approximation for a day with a uniform overcast, but with the sun's position still discernable through the cloud layer.
(d) Compute E_d and E_{od} for the sun-plus-cardioidal-sky radiance distribution of part (c).

1.9 Under what condition on the radiance $L(\vec{x};t;\theta,\phi;\lambda)$ are the horizontal components $(\vec{E})_1$ and $(\vec{E})_2$ of the vector irradiance equal to zero? When might such a radiance distribution occur in nature?

Next Chapter >>