The Radiative Transfer Chapter derived the scalar radiative transfer equation (SRTE), Eq. (3) of the Scalar Radiative Transfer Equation page:

$$\cos\theta \frac{dL(z,\theta,\phi,\lambda)}{dz} = \& - c(z,\lambda)L(z,\theta,\phi,\lambda)$$
$$+\& \int_0^{2\pi}\int_0^{\pi} \beta(z;\theta',\phi' \to \theta,\phi;\lambda)\, L(z,\theta',\phi',\lambda)\, \sin\theta' d\theta' d\phi'$$
$$+\& S(z,\theta,\phi,\lambda)\,. \tag{1}$$

This equation governs the propagation of unpolarized monochromatic radiance at a particular wavelength λ in a one-dimensional absorbing and scattering medium.

The question now arises: Is there a similar equation for the propagation of luminance? It is to be anticipated that a luminance transfer equation may be more complicated than the SRTE because it of necessity must involve all visible wavelengths and the response of the human eye.

The Luminance Transfer Equation

To develop a luminance transfer equation, multiply Eq. (1) by the photopic luminosity function $K_m\,\overline{y}(\lambda)$ and integrate over all visible wavelengths. Let Λ denote the range of wavelengths for which $\overline{y}(\lambda) > 0$. The term on the left hand side of the SRTE then becomes

$$K_m \int_\Lambda \left\{ \cos\theta \frac{dL(z,\theta,\phi,\lambda)}{dz} \right\} \overline{y}(\lambda)\, d\lambda = \cos\theta \frac{dL_v(z,\theta,\phi)}{dz}\,,$$

where the luminance L_v is defined by Eq. (1) of the Photopic Luminosity Function page:

$$L_v \equiv K_m \int_\Lambda L(\lambda)\, \overline{y}(\lambda)\, d\lambda\,.$$

The first term on the right hand side of the SRTE becomes

$$K_m \int_\Lambda \{-c(z,\lambda)\, L(z,\theta,\phi,\lambda)\}\; \overline{y}(\lambda)\, d\lambda\,.$$

This term does not give a product of an integral over wavelength of the beam attenuation coefficient times the luminance. However, we can rewrite this term as

$$\left\{ \frac{K_m \int_\Lambda -c(z,\lambda)\, L(z,\theta,\phi,\lambda)\, \overline{y}(\lambda)\, d\lambda}{K_m \int_\Lambda L(\lambda)\, \overline{y}(\lambda)\, d\lambda} \right\} K_m \int_\Lambda L(\lambda)\, \overline{y}(\lambda)\, d\lambda\,.$$

The term in braces is a radiance-weighted integral of the beam attenuation coefficient times the photopic luminosity function $\overline{y}$. If we define the photopic beam attenuation coefficient c_v as

$$c_v(z,\theta,\phi) \equiv \frac{K_m \int_\Lambda c(z,\lambda)\, L(z,\theta,\phi,\lambda)\, \overline{y}(\lambda)\, d\lambda}{K_m \int_\Lambda L(\lambda)\, \overline{y}(\lambda)\, d\lambda}\,, \tag{2}$$

then the $-c\,L$ term of the SRTE maintains the same form, $-c_v\,L_v$, in the luminance transfer equation.

A similar treatment of the path radiance term of the SRTE leads to a definition for the photopic volume scattering function:

$$\beta_v(z,\theta',\phi' \to \theta,\phi) \equiv \frac{K_m \int_\Lambda \beta(z,\theta',\phi' \to \theta,\phi,\lambda)\, L(z,\theta',\phi',\lambda)\, \overline{y}(\lambda)\, d\lambda}{K_m \int_\Lambda L(z,\theta',\phi',\lambda)\, \overline{y}(\lambda)\, d\lambda}\,. \tag{3}$$

The source term in the SRTE leads to a photopic source term:

$$S_v(z,\theta,\phi) \equiv K_m \int_\Lambda S(z,\theta,\phi,\lambda)\, \overline{y}(\lambda)\, d\lambda\,.$$

Collecting the above results gives the desired luminance transfer equation:

$$\begin{aligned}\cos\theta \frac{dL_v(z,\theta,\phi)}{dz} = \& - c_v(z,\theta,\phi)\, L_v(z,\theta,\phi) \\ +\& \int_0^{2\pi}\int_0^{\pi} \beta_v(z;\theta',\phi' \to \theta,\phi)\, L_v(z,\theta',\phi')\, \sin\theta' d\theta' d\phi' \\ +\& S_v(z,\theta,\phi)\,.\end{aligned} \tag{4}$$

This equation governs the propagation of broadband luminance as seen by the human eye in a one-dimensional absorbing and scattering medium. Equation (likesection4) is the basis for the classical definition of contrast as used in visibility studies.

Dependence of c_v on the Ambient Radiance

It is important to note that the photopic beam attenuation coefficient as defined in Eq. (likesection2) depends on the ambient radiance distribution, hence on direction (θ,ϕ), even though the beam attenuation $c(z,\lambda)$ is an inherent optical property (IOP) that does not depend on the ambient radiance or direction. Moreover, c_v meets the definition of an apparent optical property as defined on the Apparent Optical Properties page: it depends on the IOPs of the medium (here the beam attenuation c) and on the ambient radiance distribution, and it is insensitive to external conditions (e.g., rescaling L by a multiplicative factor does not change the value of c_v). The same holds true for the photopic volume scattering function defined in Eq. (likesection3) and for any other IOP. Thus, *in going from a monochromatic radiative transfer equation to a luminance transfer equation, inherent optical properties become apparent optical properties.* This is the penalty to be paid for going from an equation for monochromatic radiance as measured by instruments to an equation for luminance observed by a human eye.

However, in practice, there seems to very little dependence of c_v on the ambient radiance (as would be expected for a "good" AOP). The left panel of Fig. figure1 shows the beam attenuation $c(\lambda)$ for a simulation of homogeneous Case 1 water with a chlorophyll concentration of 0.5 $\mathrm{mg\,m^{-3}}$ (obtained using the new Case 1 IOP model in HydroLight). The Sun was at a zenith angle of $\theta_{\rm sun} = 40\,\text{deg}$ in a clear sky, which gives a transmitted solar beam of about 29 deg in the water; that beam will lie in the HydroLight quad centered at

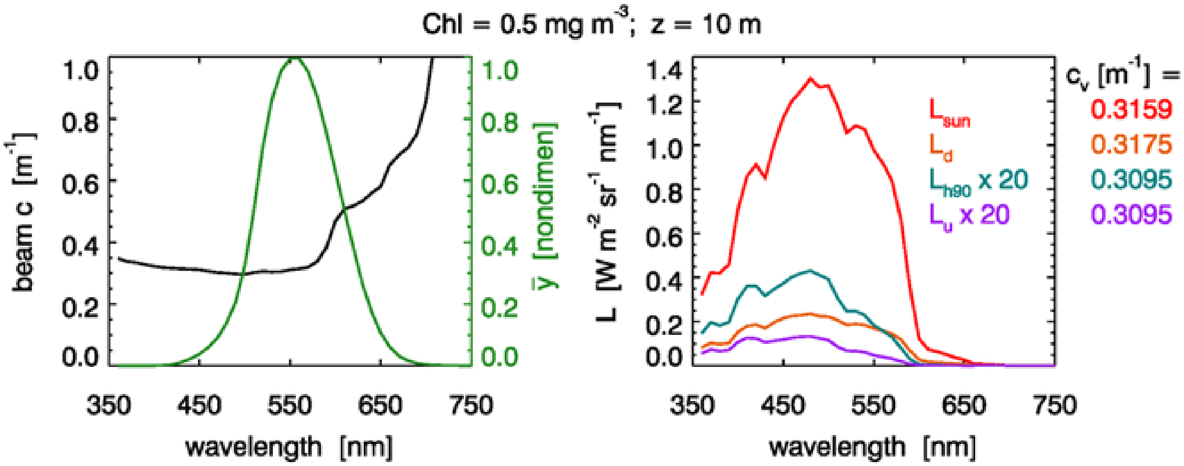

Figure 1: Left panel: Total (including water) beam attenuation $c(\lambda)$ for a chlorophyll concentration of $0.5\,\mathrm{mg\,m^{-3}}$ (black curve), and the photopic luminosity function $\bar{y}$ (green). Right panel: Radiances at a depth of 10 m for the Sun at a zenith angle of 40 deg in a clear sky. $L_{\rm sun}$ (red curve) is looking upward into the Sun's refracted beam. L_u (purple) is the upwelling (nadir-viewing) radiance; L_d (orange) is the downwelling (zenith-viewing) radiance; and L_{h90} (green) is the horizontal radiance in the direction perpendicular (azimuthal angle of $\phi = 90\,$deg) to the solar plane. L_u and L_{h90} have been multiplied by 20 for better visibility in the plot. Numbers at the right show the photopic beam attenuation c_v for the four radiance spectra.

$\theta = 30\,$deg. The right panel of Fig. figure1 shows the radiance at 10 meters depth looking in four directions: looking upward into the Sun's transmitted beam, looking in the nadir and zenith directions, and looking horizontally at right angles to the solar plane.

The spectra in this figure were used to compute the photopic beam attenuation c_v via Eq. (likesection2). The values are all close to $0.31\,\mathrm{m^{-1}}$, which is close to the beam attenuation at the peak of the photopic luminosity function: $c(555\,\mathrm{nm}) = 0.313\,\mathrm{m^{-1}}$.

Figure figure2 shows the corresponding results for a chlorophyll concentration of $10\mathrm{mg\,m^{-3}}$ and a 5 m depth. Again, the four different radiances give the same c_v to within a fraction of a percent, and these c_v values are within one percent of the beam attenuation value $c(555\,\mathrm{nm}) = 2.573\,\mathrm{m^{-1}}$.

Figure figure3 shows a chlorophyll profile consisting of a background value of $0.5\,\mathrm{mg\,m^{-3}}$ plus a Gaussian that gives a maximum value of $5.5\,\mathrm{mg\,m^{-3}}$ at 10 m depth. For this profile, an observer at 5 m depth looking upward would be looking into low-chlorophyll water, and looking downward would be looking into high-chlorophyll water. An observer at 10 m depth looking horizontally would be looking into high-chlorophyll water, but looking upward or downward would be looking into lower chlorophyll, clearer water. It might be supposed that the different IOPs ($c(z,\lambda)$ values in particular) would give radiances that might give significantly different c_v values for the different viewing directions at a given depth.

However, this is not the case. Figure figure4 shows the radiances seen by an observer at 15 m depth. Again, the c_v values differ by only about one percent from the value of

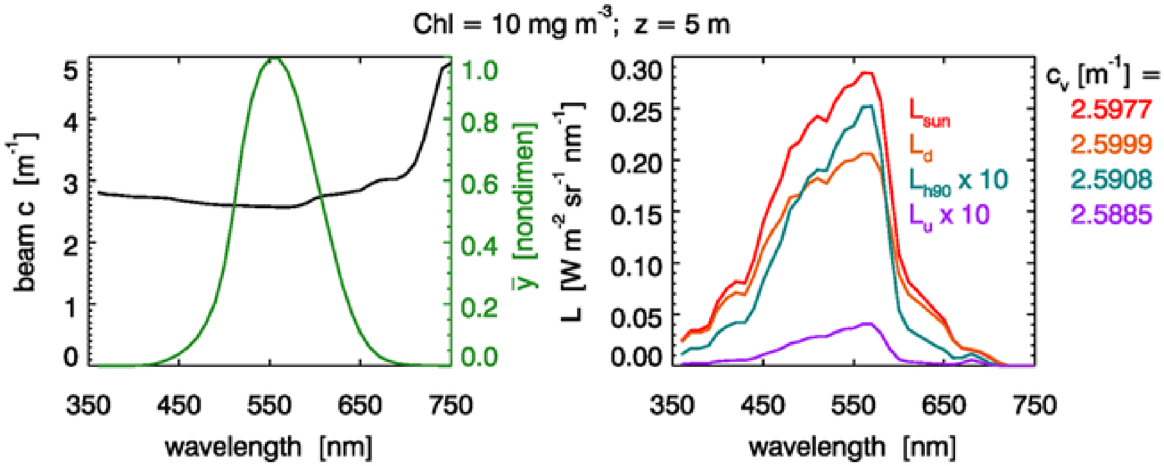

Figure 2: Same as for Fig. figure1, but for a chlorophyll concentration of 10 mg m^{-3} and a 5 m depth.

$c(15\,\text{m},\ 555\,\text{nm}) = 0.719\,\text{m}^{-1}$. The same holds true at other depths (not shown).

Exhaustive simulations have not been made, so it might be possible to create a contrived water body for which the photopic beam attenuation would be significantly different for different viewing directions, and for which $c_v(z)$ would be significantly different from $c(z, 555\,\text{nm})$. However, the above simulations indicate that in many situations of practical interest, there is little dependence of c_v on viewing direction, and that c_v is within a percent or so of the beam attenuation at the 555 nm wavelength of the maximum of the photopic luminosity function.

These simulations are consistent with the results of Zaneveld and Pegau (2003), who found that the beam attenuation coefficient at 532 nm (excluding the water contribution) is a good proxy for the photopic beam attenuation.

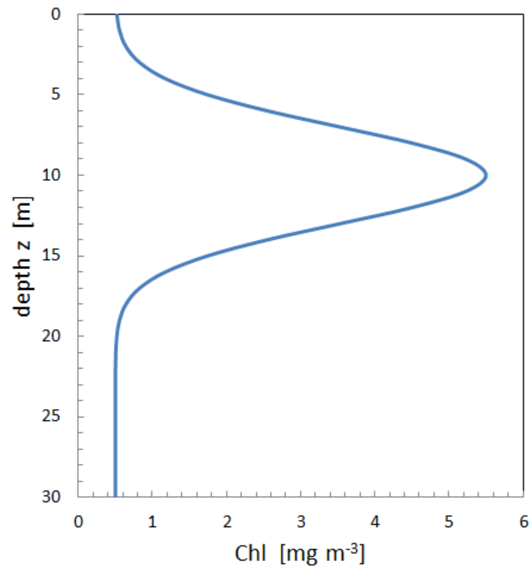

Figure 3: The chlorophyll profile used in the simulations of Fig. figure4.

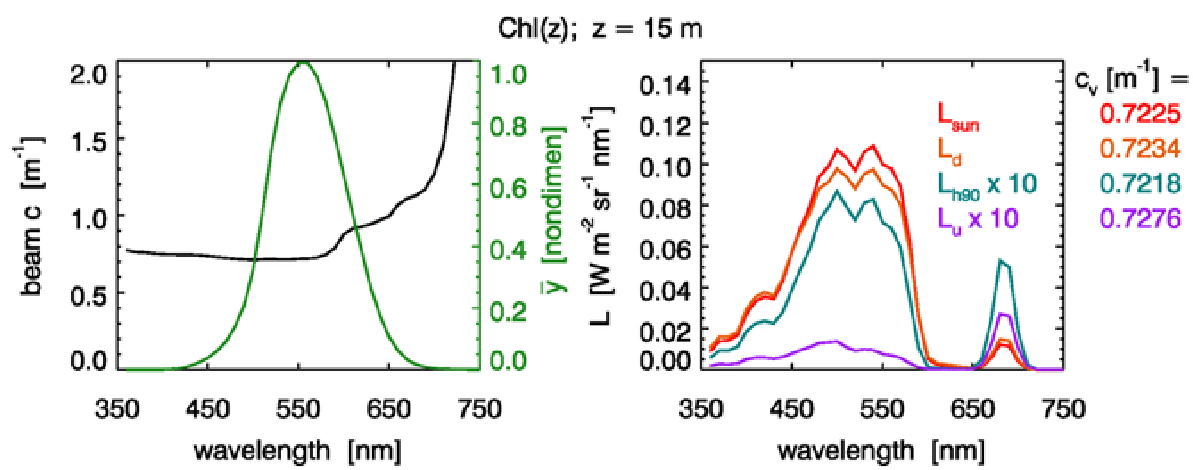

Figure 4: Same as for Fig. figure1, but for the depth-dependent chlorophyll profile seen in Fig. figure3 and a 15 m depth.